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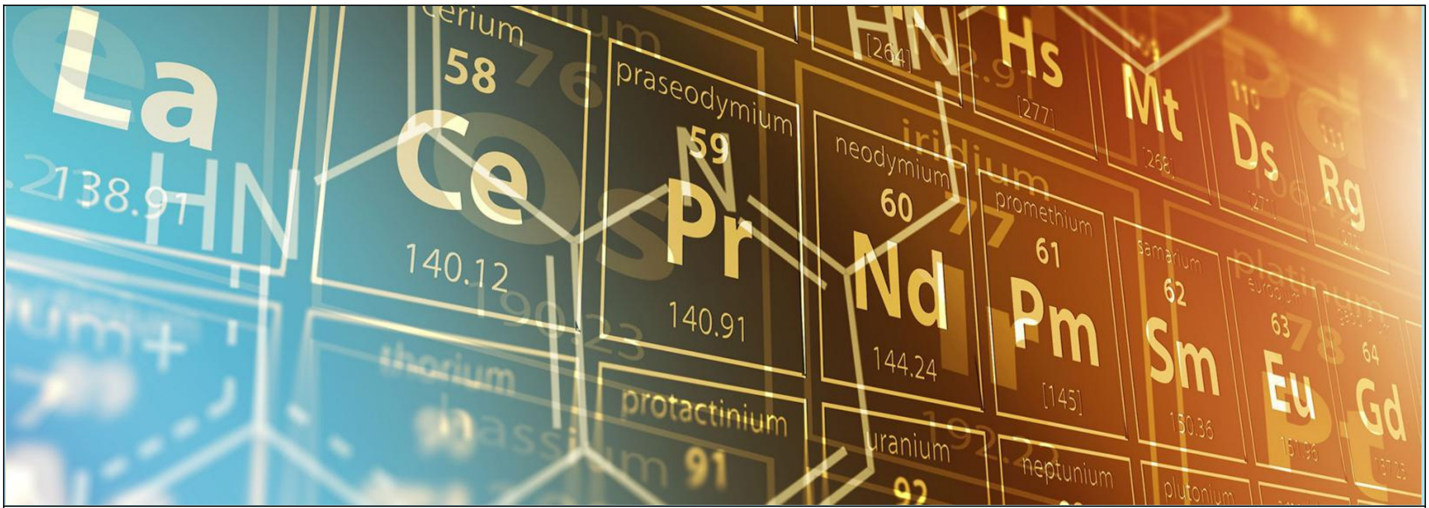

Longitudinal Form Factors with Core- Polarization Effects for ^{6}Li and ^{13}C nuclei

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ABSTRACT

Inelastic longitudinal electron scattering form factors C2 transitions have been studied in ^{6}Li and ^{13}C nuclei with the aid of shell model calculations. The core polarization transition density was evaluated by adopting the shape of Tassie model togther with the derived form of the ground state two-body charge density distributions (2BCDD's). The following transitions have been investigated; $1^{+}0 \rightarrow 3^{+}0$ of ^{6}Li and $1/2^{-}1/2 \rightarrow 3/2^{-}, 1/2$ of ^{13}C nuclei. It is found that the core polarization effects, which represent the collective modes, are essential for reproducing a remarkable agreement between the calculated inelastic longitudinal C2 form factors and those of experimental data.

KEYWORDS: Inelastic longitudinal form factors, two-body charge density and core polarization effects.

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I. INTRODUCTION

The scattering of electrons from nuclei gives the most precise information about nuclear size and charge distribution, and it provides important information about the electromagnetic currents inside the nuclei. Electron scattering can provide a good test for such calculation since it is sensitive to the spatial dependence of the charge and current densities [1].

The electron scattering from the nucleus at high energy gives an important information about the nuclear structure. The obtained information from the high energy electron scattering by the nuclei depends on the magnitude of the de Broglie wavelength that is associated with the electron compared with the range of the nuclear forces. When the energy of the incident electron is in the range of 100 *MeV* and more, the de Broglie wavelength will be in the range of the spatial extension of the target nucleus. Thus with these energies, the electron represents a best probe to study the nuclear structure [2]. Shell model within a restricted model space is one of the models, which succeeded in describing static properties of nuclei, when effective charges are used. Calculations of form factors using the model space wave function alone is inadequate for reproducing the data of electron scattering [3]. Therefore, effects out of the model space, which are called core polarization effects, are necessary to be included in the calculations. Various theoretical methods [4]-[6] Therefore effects out of the model space, which is called core polarization effects, are necessary to be included in the calculations. Core polarization effects can be treated either by connecting the ground state to the *J*-multipole $n\hbar\omega$ giant resonances [7], where the shape of the transition densities for these excitations is given by the Tassie model [8]or by using a microscopic theory [9]-[13] which permits one particle-one hole (1p-1h) excitations of the core and also of the model space to describe these longitudinal excitations. Comparisons between theoretical and observed longitudinal electron scattering form factors have long been used as stringent test of models of nuclear structure.

Elastic and inelastic electron scattering form factors for the 1p-shell nuclei in the frame work of the many-particle shell model with included in the core-polarization effects [14]-[17]

The aim of the present work is to study the inelastic longitudinal form factor C2 for ^{6}Li and ^{13}C nuclei. The calculation of form factors using the many particle shell model space alone were known to be inadequate in describing electron scattering data. So effects out of the model space (core-polarization) are necessary to be included in the calculations. The shape of the transition density for the excitation considered in this work was given by the Tassie model [8], where this model is connected with the ground state charge density, where the ground state charge density of the present work is to derive an expression for the ground state two - body charge density distributions (2BCDD's), based on the use of the two - body wave functions of the harmonic oscillator and the full two–body correlation functions FC's(which include the tensor correlations TC's and short range correlations SRC's).

II. THEORY

The interaction of the electron with charge distribution of the nucleus gives rise to the longitudinal or Coulomb scattering. The longitudinal form factor is related to the charge density distributions (CDD) through the matrix elements of multipl operators $\hat{T}_J^L(q)$ [7]

$$\left| F_J^L(q) \right|^2 = \frac{4\pi}{Z^2(2J_i+1)} \left| \left\langle f \left\| \hat{T}_J^L(q) \right\| i \right\rangle \right|^2 \left|F_{cm}(q)\right|^2 \left|F_{fs}(q)\right|^2 \quad .....(1)$$

Where Z is the atomic number of the nucleus, $F_{cm}(q)$ is the center of mass correction, which remove the spurious state arising from the motion of the center of mass when shell model wave function is used and given by [7].

$$F_{cm}(q) = e^{q^2b^2/4A} \quad ....(2)$$

Where A is the nuclear mass number and b is the harmonic oscillator size parameter. The function $F_{fs}(q)$ is the free nucleon form factor and assumed to be the same for protons and neutrons and takes the form [18].

$$F_{fs}(q) = \left[1 + \left(\frac{q}{4.33} \right)^2 \right]^{-2} \quad ......(3)$$

The longitudinal operator is defined as [19].

$$\hat{T}_{Jt_z}^L(q) = \int dr\; j_J(qr) Y_J(\Omega)\, \rho(r,t_z) \quad ....(4)$$

Where $j_J(qr)$ is the spherical Bessel function, $Y_J(\Omega)$ is the spherical harmonic wave function and $\rho(r,t_z)$ is the charge density operator. The reduced matrix elements in spin and isospin space of the longitudinal operator between the final and initial many particles states of the system including the configuration mixing are given in terms of OBDM elements times the single particle matrix elements of the longitudinal operator [7],i.e.

$$\left\langle f \left\| \hat{T}^L_{JT} \right\| i \right\rangle = \sum_{a,b} OBDM^{JT}(i,f,J,a,b) \left\langle b \left\| \hat{T}^L_{JT} \right\| a \right\rangle \quad .....(5)$$

The many particle reduced matrix elements of the longitudinal operator, consists of two parts one is for the model space and the other is for core polarization matrix element [6].

$$\left\langle f \left\| \hat{T}^L_J(\tau_Z,q) \right\| i \right\rangle = \left\langle f \left\| \overset{ms}{\hat{T}^L_J}(\tau_Z,q) \right\| i \right\rangle + \left\langle f \left\| \overset{cor}{\hat{T}^L_J}(\tau_Z,q) \right\| i \right\rangle \quad .....(6)$$

Where the model space matrix element in Eq.(6) has the form [7].

$$\left\langle f \left\| \overset{ms}{\hat{T}^L_J}(\tau_Z,q) \right\| i \right\rangle = e_i \int_0^\infty d\mathrm{r}\, \mathrm{r}^2 j_J(q\mathrm{r}) \overset{ms}{\rho}_{J,\tau_Z}(i,f,\mathrm{r}) \quad .....(7)$$

Where $\overset{ms}{\rho}_J(i,f,\mathrm{r})$ is the transition charge density of model space and given by [7].

$$\overset{ms}{\rho}_{J,\tau_Z}(i,f,\mathrm{r}) = \sum_{jj'(ms)} OBDM(i,f,J,j,j',\tau_z) \left\langle j \| Y_J \| j' \right\rangle R_{nl}(\mathrm{r}) R_{n'l'}(\mathrm{r}) \quad ....(8)$$

The core polarization matrix element is given by [7]

$$\left\langle f \left\| \overset{cor}{\hat{T}^L_J}(\tau_Z,q) \right\| i \right\rangle = e_i \int_0^\infty d\mathrm{r}\, \mathrm{r}^2 j_J(q\mathrm{r}) \overset{core}{\rho_J}(i,f,\mathrm{r}) \quad ......(9)$$

Where $\overset{core}{\rho_J}$ is the core polarization transition density which depends on the model used for core polarization. To take the core polarization effects into consideration, the model space transition density is added to the core polarization transition density that describes the collective modes of nuclei. The total transition density becomes

$$\rho_{J\tau_Z}(i,f,r) = \overset{ms}{\rho}_{J\tau_Z}(i,f,r) + \overset{core}{\rho}_{J\tau_Z}(i,f,r) \quad ....(10)$$

Where $\overset{core}{\rho}_J$ is assumed to have the form of Tassie shape and given by [8].

$$\rho^{core}_{Jt_z}(i,f,r) = N\frac{1}{2}(1+\tau_z) r^{J-1} \frac{d\rho(i,f,r)}{dr} \quad ......(11)$$

Where N is a proportionality constant. $\rho(i,f,r)$ is the ground state charge density distribution. It is derived an effective two-body charge density operator (to be used with uncorrelated wave functions) can be produced by folding the two-body charge density operator with the two-body correlation functions $\widetilde{f}_{ij}$ as [20]

$$\hat{\rho}^{(2)}_{eff}(\vec{\mathrm{r}}) = \frac{\sqrt{2}}{2(A-1)} \sum_{i\neq j} \widetilde{f}_{ij} \left\{ \delta\left[\sqrt{2}\,\vec{\mathrm{r}} - \vec{R}_{ij} - \vec{\mathrm{r}}_{ij}\right] + \delta\left[\sqrt{2}\,\vec{\mathrm{r}} - \vec{R}_{ij} + \vec{\mathrm{r}}_{ij}\right] \right\} \widetilde{f}_{ij} \quad ...(12)$$

Where $\vec{\mathrm{r}}_{ij}$ and $\vec{R}_{ij}$ of relative and center of mass coordinates and the form of $\widetilde{f}_{ij}$ is given by [23].

$$\widetilde{f}_{ij} = f(r_{ij})\,\Delta_1 + f(r_{ij})\left\{1 + \alpha(A)\,S_{ij}\right\}\Delta_2 \quad ...(13)$$

It is clear that Eq. (13) contains two types of correlations:

1. The two body short range correlations presented in the first term of Eq. (13) and denoted by $f(r_{ij})$. Here Δ_1 is a projection operator onto the space of all two-body functions with the exception of $^3S_1$ and $^1D_3$ states. It should be remarked that the short range correlations are central functions of the separation between the pair of particles which reduce the two-body wave function at short distances,where the repulsive core forces the particles apart, and heal to unity at large distance where the interactions are extremely weak. A simple model form of $f(r_{ij})$ is given as [21]

$$f(r_{ij}) = \begin{Bmatrix} 0 & for\ r_{ij} \le r_c \\ 1-\exp\{-\mu(r_{ij}-r_c)^2\} & for\ r_{ij} > r_c \end{Bmatrix} \quad ....(14)$$

Where r_c =0.4(in fm) is the radius of a suitable hard core and $\mu = 25\ fm^{-2}$ [20] is a correlation parameter.

2. The two-body tensor correlations presented in the second term of Eq.(13) are induced by the strong tensor component in the nucleon-nucleon force and they are of longer range. Here Δ_2 is a projection operator onto $^1S_3$ and $^1D_3$ states only. S_{ij} is the usual tensor operator, formed by the scalar product of a second-rank operator in intrinsic spin space and coordinate space and is defined by

$$S_{ij} = \frac{3}{r_{ij}^2}(\vec{\sigma}_i.\vec{r}_{ij})(\vec{\sigma}_j.\vec{r}_{ij}) - \vec{\sigma}_i.\vec{\sigma}_j \quad ......(15)$$

The parameter $\alpha(A)$ is the strength of tensor correlations and it is non zero only in the ${}^1S_3 - {}^1D_3$ channels.
The Coulomb form factor for this model becomes,

$$F_J^L(q) = \sqrt{\frac{4\pi}{2J_i+1}}\,\frac{1}{Z}\left\{\int_0^\infty \mathrm{r}^2 j_J(q\mathrm{r})\,\rho_J^{ms}(i,f,\mathrm{r})\,d\mathrm{r} + \mathrm{N}\int_0^\infty d\,\mathrm{r}\,\mathrm{r}^2 j_J(q\mathrm{r})\,\mathrm{r}^{J-1}\frac{d\rho_o(i,f,\mathrm{r})}{d\mathrm{r}}\right\} F_{cm}(q)\,F_{fs}(q) \quad \text{....(16)}$$

The radial integral $\int_0^\infty d\mathrm{r}\ \mathrm{r}^{J+1} j_J(q\mathrm{r})\frac{d\rho_o(i,f,\mathrm{r})}{d\mathrm{r}}$ can be written as:-

$$\int_0^\infty \frac{d}{d\mathrm{r}}\left\{\mathrm{r}^{J+1}\ j_J(q\mathrm{r})\,\rho_o(i,f,\mathrm{r})\right\}d\mathrm{r} - \int_0^\infty d\mathrm{r}\ (J+1)\,\mathrm{r}^J\ j_J(q\mathrm{r})\,\rho_o(i,f,\mathrm{r})$$
$$-\int_0^\infty d\mathrm{r}\ \mathrm{r}^{J+1}\frac{d}{d\mathrm{r}}\,j_J(q\mathrm{r})\,\rho_o(i,f,\mathrm{r})$$

Where the first term gives zero contribution, the second and the third term can be combined together as

$$-q\int_0^\infty d\mathrm{r}\ \mathrm{r}^{J+1}\rho_o(i,f,\mathrm{r})\left[\frac{d}{d(q\mathrm{r})} + \frac{J+1}{q\mathrm{r}}\right] j_J(q\mathrm{r}) \quad \text{.......(17)}$$

From the recurssion relation of spherical Bessel function:

$$\left[\frac{d}{d(q\mathrm{r})} + \frac{J+1}{q\mathrm{r}}\right] j_J(q\mathrm{r}) = j_{J-1}(q\mathrm{r}) \quad \text{.........(18)}$$

$$\therefore \int_0^\infty d\mathrm{r}\ \mathrm{r}^{J+1} j_J(q\mathrm{r})\frac{d\rho_o(i,f,\mathrm{r})}{d\mathrm{r}} = -q\int_0^\infty d\mathrm{r}\,\mathrm{r}^{J+1}\ j_{J-1}(q\mathrm{r})\,\rho_o(i,f,\mathrm{r}) \quad \text{.......(19)}$$

hence, the form factor of Eq. (16) takes the form

$$F_J^L(q) = \left(\frac{4\pi}{2J_i+1}\right)^{1/2}\frac{1}{Z}\left\{\int_0^\infty \mathrm{r}^2 j_J(q\mathrm{r})\,\rho_{Jt_z}^{ms}\,d\mathrm{r} - \mathrm{N}q\int_0^\infty d\mathrm{r}\,\mathrm{r}^{J+1}\rho_o(i,f,r)\,j_{J-1}(q\mathrm{r})\right\}\times F_{cm}(q)\,F_{fs}(q) \quad \text{....(20)}$$

The proportionality constant N can be determined from the form factor evaluated at *q*=*k*, i.e., substituting *q*=*k* in Eq.(20), we obtain

$$N = \frac{\int_0^\infty d r\, r^2 j_J(k r)\, \rho_{Jt_Z}^{ms}(i,f,r) - F_J^L(k)\, Z \sqrt{\frac{2J_i+1}{4\pi}}}{k\int_0^\infty d r\, r^{J+1} \rho_o(i,f,r)\, j_{J-1}(k r)} \quad \ldots\ldots\ldots\ldots(21)$$

The reduced transition probability $B(CJ)$ is written in terms of the form factor in the limit $q = k$ (photon point) as [7].

$$B(CJ) = \frac{[(2J+1)!!]^2 Z^2 e^2}{4\pi\, k^{2J}} \left| F_J^L(k) \right|^2 \quad \ldots\ldots\ldots \quad (22)$$

In Eq.(21), the form factor at the photon point q=k is related to the transition strength $B(CJ)$. Thus using Eq.(22) in Eq.(21) leads to

$$N = \frac{\int_0^\infty d r\, r^{J+2} \rho_{Jt_Z}^{ms}(i,f,r) - \sqrt{(2J_i+1) B(CJ)}}{(2J+1)\int_0^\infty d r\, r^{2J} \rho_o(i,f,r)} \quad \ldots\ldots\ldots \quad (23)$$

The proportionality constant N can be determined by introducing the experimental value of the reduced transition probability $B(CJ)$ into eq.(23).

III. RESULTS AND DISCUSSION

The inelastic longitudinal electron scattering form factors C2 are calculated using an expression for the transition charge density of Eq.(10). The model space transition density is obtained using Eq.(8). For considering the collective modes of the nuclei, the core polarization transition density of Eq.(11) were evaluated by adopting the Tassie model together with the calculated ground state 2BCDD of Eq.(13).

All parameters required in the following calculations of 2BCDD's, $\langle r^2 \rangle^{1/2}$ and longitudinal $F(q)$'s are presented in Tables (1).

Table 1. Parameters which have been used in the present calculations for the 2BCDD's, $\langle r^2 \rangle^{1/2}$ and elastic and inelastic longitudinal F(q)'s of all nuclei under study .

Nucleus	***b*** (fm)	$\alpha(A)$	$\langle r^2 \rangle^{1/2}_{Theo.}$ (fm)	$\langle r^2 \rangle^{1/2}_{Exp.}$ (fm)[22]
^{6}Li	1.650	0.075	2.100	2.260
^{13}C	1.700	0.080	2.563	2.400

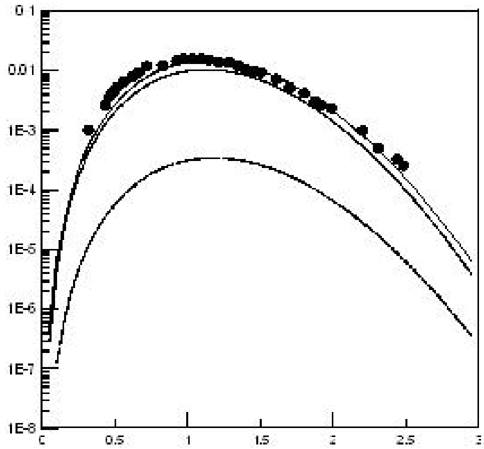
3.1. The nucleus ^6Li

The nucleus is excited by the incident electron from the ground state ($J_i^{\pi}T_i$ = 1⁺0) to the state $J_f^{\pi}T_f$ =3⁺0 with an excitation energy of 2.18 MeV. The longitudinal C2 form factor is of isoscalar character. A comparison between the experimental and theoretical form factors for the C2 transition for ^{6}Li is given in Fig.(1) shows the relation between the longitudinal Coulomb C2 electron scattering form factors as a function of momentum transfers. The dashed curves represent the contribution of the model space, the dashed-dotted curve corresponding to the result of core polarization effects and the solid curves represent the total contribution, which is obtained by taking the model space together with the core polarization effects where the effect of two-body SRC's and TC's are considered, and the dotted symbols represent the experimental data Ref. [22] . The model space calculations understimate the the experimental data for the region of momentum transfer. The core polarization effects is added to the model space, the obtained results for the longitudinal *C2* form factors become resonbal accordance with those of experimental data throughout the whole range of momentum transfer *q*.

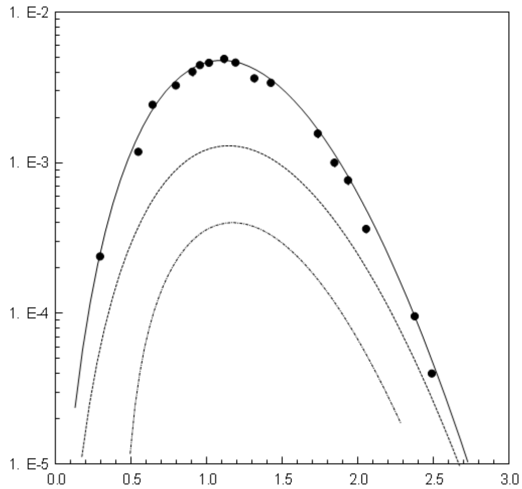
3.2 The nucleus ^{13}C

The calculation is performed for the isoscalar C2 transition from the ground state ($J_i^{\pi}T_i$ =1/2⁻ 1/2) to the excited state $J_f^{\pi}T_f$ =3/2⁻, 1/2 by the incident electron with an excitation energy of 3.68 MeV.The transition under investigation is C2, 3.68 MeV structure and properties of ^{13}C are experimentally and theoretically well studied. In figure (2) the experimental data of the C2 Coulomb form factors which are taken from Ref. [23]are compared with the theoretical shell model calculation. The dashed curves represent the contribution of the model space, the dashed-dotted curve corresponding to the result of core polarization effects and the solid curves represent the total contribution, which is obtained by taking the model space together with the core polarization effects where the effect of two-body SRC's and TC's are considered, and the dotted symbols represent the experimental data Ref. [23]. The 1p-shell fail to describe the data in form factors. Core polarization effects enhance the form factor and reproduce the measured form factor as shown by solid curve of figure (2).

Figure(1): Inelastic longitudinal form factors for the transition to the 3⁺ in the ^{6}Li with and without core-polarization effects and by using Tassie model , the experimental data are taken from Ref. [22]



Figure(2): Inelastic longitudinal form factors for the transition to the 2⁺ in the ^{13}C with and without core-polarization effects and by using Tassie model, the experimental data are taken from Ref. [23]



IV. CONCLUSIONS

1) The effect of FC's is, generally, essential in getting good agreement between the calculated results of $\langle r^2 \rangle^{1/2}$ and those of experimental data.
2) The 1p-shell models, which can describe the static properties and energy levels, are less successful for describing dynamics properties such as C2 transitions rates and electron scattering form factors.
3) The core-polarization effect enhances the form factors and makes the theoretical results of the longitudinal form factors closer to the experimental data in the C2 transitions which are studied in the present work.

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