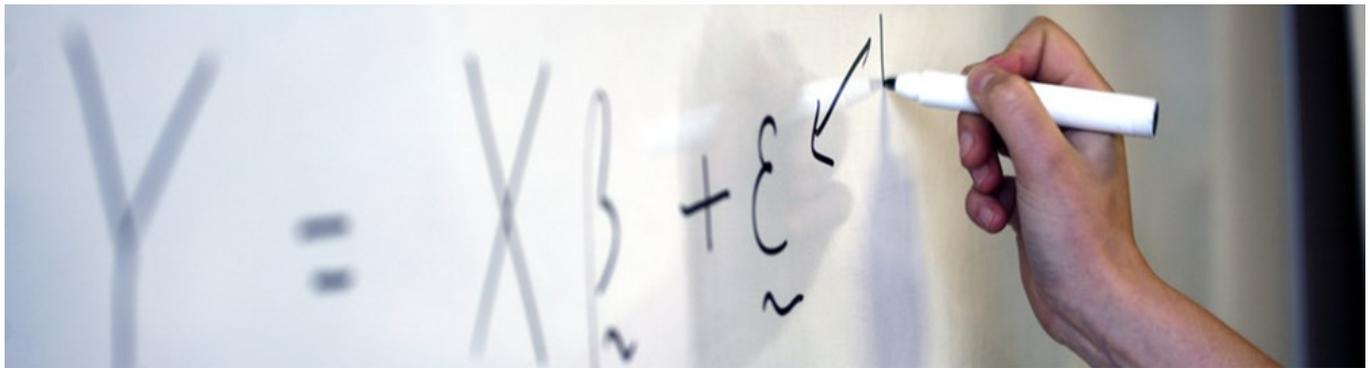


# DUAL SEARCH FIREFLY ALGORITHM FOR FUNCTION OPTIMIZATION



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## ABSTRACT

Accurate forecast of the micrometric precision of function value are extremely important in the optimization process. It reduces operation cost and gives accurate control of process quality. Due to the uncertain characteristics of the complicated function, many research studies have moved to nonlinear forecasting methods. One of the forecasting methods used in the mentioned issue is firefly algorithm (FA). The FA is a meta-heuristic algorithm that has been successfully applied in many practical applications. However, the FA had a few disadvantages in global searching including slow convergence speed, high possibility of being trapped in local optimum and low solution accuracy. Therefore, this paper proposed an improved FA known as dual search firefly algorithm (DSFA), for solving the function optimization problems. The DSFA employed the characteristics of speedy local search approach with the global optimization of FA. The experimental results showed that the proposed algorithm has overall improved over 85% in performances compared to FA in terms of convergence speed and accuracy.

**KEYWORD:** Function approximation, optimization, firefly algorithm, local optimum, prediction.

## I. INTRODUCTION

Today, science and technology and information technology have developed rapidly. The development level and quality of life of human society have taken a huge leap. With the deepening of research on natural science, information science and engineering technology, science and technology are in a variety of disciplines intercross, resulting in many new research areas, research methods, and the corresponding new technologies have constantly advanced to accelerate the development of civilization.

In scientific research, finance, economic management, engineering design, national defense and so on, we often encounter the most reasonable solution from a variety of schemes. This is an optimization problem. Therefore, it can be summarized as follows: under certain constraints, from all possible solutions, select some performance variables that can make the problem to achieve the maximum or minimum scheme.

Optimization problem is an ancient problem based on mathematics. Many researchers have study and explore the optimization technology that has produced a great economic and social benefit in various fields. The conventional optimization method includes the conjugate gradient method, simplex method, pattern search method, Lagrange multiplier method, Newton method and Powell method. These algorithms have played an important role, while solving practical problems, they still consists of some limitations.

While constantly improving the conventional optimization algorithm, the Darwin's theory of evolution and phenomenon of natural life has inspired researchers; they discover that the nature of each species has certain group behavior. According to these behaviors, the swarm intelligence optimization algorithm has been proposed. J. Holland from the genetic mechanism of biological populations department, After series of observation and study in genetics, on the rule of survival of the fittest, the first genetic algorithm (GA) optimization algorithm was proposed (Holland, 1975). Also proposed is the ant colony optimization algorithm, inspired by the ants behavior in finding the path and transmission of information for food (Colorni, Dorigo, & Maniezzo, 1991). The particle swarm optimization algorithm based on the flocking behavior of birds was studied (Kennedy & Eberhart, 1995). The artificial fish swarm algorithm to solve large-scale reliability-redundancy application problem was proposed (He, Hu, Ren, & Zhang, 2015).

In recent decades, many intelligence optimization algorithms have developed rapidly. Compared with conventional optimization methods, it has the characteristic of easy realization, simple in theory, and superior optimization effect. Hence, due to its efficiency in solving the problem of optimization without strict constraints, the swarm intelligence algorithm has been applied in various fields. For instance, (Chandrasekaran & Simon, 2012) training neural network, optimize intelligent robot movement (Laskari, Parsopoulos, & Vrahatis, 2002), scheduling problems (Ji, Sun, Dui, & Ren, 2017), integer programming (Verwer, Zhang, & Ye, 2015) and searching shortest path (Faro & Giordano, 2016).

FA is a new heuristic intelligent optimization method (X.-S. Yang, 2009). The basic idea comes from the biological characteristics of firefly adults using light-emitting biological characteristics. The algorithm is based on the location of fireflies to describe the individual firefly brightness and the degree of attraction to other fireflies. The higher the brightness of the firefly, the better their position, and the greater their degree of attraction. Each firefly is updated according to the brightness and the degree of attraction of its neighboring fireflies, thus achieving the purpose of position optimization.

The FA has been proposed by many researchers, and has been successfully applied to combinatorial optimization (Sayadi, Ramezani, & G. N., 2010), path planning (Srivatsava, Mallikarjun, & Yang, 2013), image processing

(Hornig & Liou, 2011), economic scheduling (Udaiyakumar & Chandrasekaran, 2014) and other fields, showing a good optimization performance and practical application potential. However, the FA has just been proposed in recent years as one of the new swarm intelligence optimization algorithm. There are many deficiencies for this algorithm, such as easily trap in local optimal and slow convergence speed, and its scope of application is still small. So, further research and analysis of the rules and performance is necessary to expand its application field.

A Firefly Algorithm (X.-S. Yang, 2009) when used on ten standard test functions, shows that FA is superior to Particle Swarm Optimization (PSO) and GA from the average number of valuation and success rate, and had better solving for NP-hard problem (X.-S. Yang, 2009). A hybrid optimization algorithm was proposed (X. S. Yang & Deb, 2010) by combining Levy walk and FA for solving global optimization problem of the optimization algorithm, called Eagle Strategy (Xin-She Yang, 2010). A multilevel image threshold scheme based on FA was introduced (Hornig & Liou, 2011).

FA was used for clustering approaches and evaluation of its performance, indicating that the FA is an effective, reliable and robust method, that can used successfully in finding cluster centers (Senthilnath, Omark, & Mani, 2011). Furthermore, FA was applied to optimize the six mechanical structure designs for telescopic rope design (Amir Hossein Gandomi, Yang, & Alavi, 2011). Gaussian distribution was used to enhanced FA convergence speed in solving global optimum problem (S.M. Farahani, A. Abshouri, B. Nasiri, M. Meybodi, 2012).

A non-parallel FA solving a constrained optimization problem was proposed. A hybrid evolutionary FA, which integrates the differential evolution algorithm into the FA, that improve the searching accuracy and information sharing between fireflies for finding local optimum value (Abdullah, Deris, Mohamad, & Hashim, 2012). Additionally, a hybrid model which combine chaotic FA and the support vector regression machine for the stock-market price forecast (Kazem, Sharifi, Hussain, Saberi, & Hussain, 2013).

There are three ways to improve basic fireflies (S.M. Farahani, A. Abshouri, B. Nasiri, M. Meybodi, 2012). The first one is to use learning automata to influence absorption factors and randomization parameters, and second is hybrid of GA and FA, third is random moving fireflies in search space based on the Gauss distribution. Moreover, the FA was applied on task scheduling optimization problem (X. S. Yang, Hosseini, & Gandomi, 2012). The different chaotic maps for standard function optimization was analyzed using FA (A. H. Gandomi, Yang, Talatahari, & Alavi, 2013). In addition, the effects of different chaotic maps by optimizing the standard functions using FA and extended the application to search for discrete independent path in maps were studied (Srivatsava et al., 2013).

The optimization mechanism of FA from the mathematical point of view was described, through the application in multilevel inverter with adjustable DC sources, shows the feasibility and effectiveness of FA optimization (Gnana Sundari, Rajaram, & Balaraman, 2016). The FA was used for tracing power leakage for nanoscale CMOS circuit for improving battery life of the system (Kougianos & Mohanty, 2015). The use of FA was reported in predicting the total electron content of seismo-ionospheric anomalies for earthquake detection (Akhoondzadeh, 2015).

The FA was used in heart disease prediction by improving the initial selection of center point for feeding into C-means clustering algorithm (Long, Meesad, & Unger, 2015). The FA approach was improved to solved chiller loading energy conservation problem, and the results showed the FA ability to minimize the energy consumption for chiller (Dos Santos Coelho & Mariani, 2013). The FA was proposed for solving power

system optimal reactive power dispatch problems (Rajan & Malakar, 2015). These approaches are to improve the slow convergence rate and easy trapping in local extreme value region of FA. It uses the random initial point for calculating the reflection, expansion and contraction point values. This proposed hybrid FA was reported to have had better convergence characteristics and robustness compared to conventional FA and other existing methods.

A hybrid firefly-GA (Rahmani & MirHassani, 2014), that combines the strong global search ability of FA and the strong local search characteristic of GA, for capacitated facility location problem. This solved the FA problem in trapping into local minima, and the mutation ability in GA are applied to improve the fireflies location for attraction process. The FA was used in continuous space optimization to managed congestion in deregulated environment (Verma, Saha, & Mukherjee, 2016). A chaotic FA was introduced through the tent map function, that dynamically shrink the search space to speed up the convergence rate (Gokhale & Kale, 2016).

Consequently, similar to the other optimization algorithms, the FA also has slower convergence rate and trapping in local optima. Hence, the main purpose of this paper is to improve the performance of FA by implementing a modified simplex method and to solve surface roughness problem in machining process. To verify the effectiveness of the proposed approach, the approach was tested with standard function and function with constraint (Anon, 2017), and the surface roughness data were tested. In this work, the proposed method is discussed in details and the simulation results are compared with various benchmark function results. The results from experiment and simulations show that the proposed approach out-performed FA and displayed its robustness in solving surface roughness problem.

**II. FIREFLY ALGORITHM (FA)**

FA was first proposed by ((X.-S. Yang, 2009; Xin-She Yang, 2014)). This algorithm is inspired by fireflies rhythmic flashes behaviors that give bioluminescence. Furthermore, it is reported that the flashing pattern is used to attract partners, or potential prey. Hence, this light intensity of fireflies is the main factor causing other fireflies to move towards each other. To simplify the algorithm, three idealized rules are considered. First, all the fireflies are unisex, so this means that these fireflies are attracted to each other irrespective of their sex. Second, the attractiveness and brightness of fireflies are proportional to each other, meaning that the brighter one attracts the less bright one. Attractiveness and brightness are both decreased with the increases in distance. Furthermore, in case there is no one brighter to other firefly, then random movements are considered. Third, the brightness of fireflies is determined as objective function value.

In the FA, the light intensity and the attractiveness are two important issues. For simplicity, we assume that the firefly attractiveness is determined by its brightness. In the maximization problems, the brightness,  $I$ , of firefly on particular location,  $X$ , are formulated as  $I(X) = f(X)$ . Despite that the attractiveness  $\beta$  is relative, however, it should judge by other fireflies. Hence, it will vary with the distance between firefly- $i$  and firefly- $j$ . Furthermore, the brightness decreases with distance from its source, and it can also be absorbed by the medium from the surrounding. So, the attractiveness varies with the varying degree of absorption. The brightness in simplest form can be formulated as follows:

$$I(r) = I_0 e^{-\gamma r_{ij}^2} \dots\dots\dots (2.1)$$

where  $I_0$  is the original light intensity,  $\gamma$  is the light absorption coefficient, and  $r_{ij}$  is the distance between firefly- $i$  and firefly- $j$ . In addition,  $r_{ij}$  can formulate as follows:

$$r_{ij} = \|X_i - X_j\| = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \dots\dots\dots (2.2)$$

The firefly attractiveness is proportional to light intensity determined by adjacent fireflies. It can mathematically be expressed as the following:

$$\beta = \beta_{min} + (\beta_0 - \beta_{min}) e^{-\gamma r_{ij}^2} \dots\dots\dots (2.3)$$

where  $\beta_0$  is the attractiveness at  $r = 0$ , and  $\beta_{min}$  represent the minimum value of attractiveness. As mentioned above, firefly- $i$  is attracted to a brighter firefly- $j$ , and the movement of the  $i$ -th firefly can be formulated as follows:

$$x_i = x_i + \beta(x_i - x_j) + \alpha(rand - 0.5) \dots\dots\dots (2.4)$$

where  $\alpha$  is the randomization parameter and  $rand$  is the uniformly generated number in range  $[0, 1]$ . All setting for the constant values is listed in Table I.

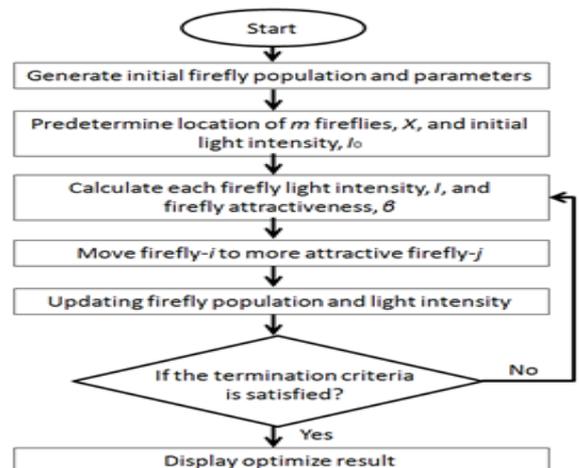
**Table I: The initial reference value for each parameter of FA**

$\alpha$	$\beta_0$	$\beta_{min}$	$\gamma$
0.2	1.0	0.2	1.0

**A. The Steps of FA**

The basic steps involved in FA is shown in Figure 1 and described as follows:

- Step 1:** Initialize FA parameters such as, initial firefly population,  $m$ , randomization parameter,  $\alpha$ , initial attractiveness,  $\beta_0$ , minimum value of attractiveness,  $\beta_{min}$ , light absorption coefficient,  $\gamma$ , and maximum iteration.
- Step 2:** Predetermine the location of  $m$  fireflies at  $X_i$  ( $i = 1, 2, \dots, m$ ), and Initial light intensity for each firefly,  $I_0$ .
- Step 3:** Calculate each firefly light intensity,  $I$ , and firefly attractiveness,  $\beta$ , using Equation (2.1) and Equation (2.3).
- Step 4:** Move firefly- $i$  to more attractive firefly- $j$  using Equation (2.4).
- Step 5:** Updating firefly location and light intensity.
- Step 6:** Determine whether the termination criteria are satisfied. If it is fulfilled, move to Step 7, else move to Step 3 and continue the flow again.
- Step 7:** Display optimize result.



**Figure 1. The flow chart of FA**

**III. DUAL SEARCH FIREFLY ALGORITHM (DSFA)**

Simplex method proposed by (Nelder & Mead, 1964) is one of the optimization method that depends on comparing the objective function values at the vertices. This method shows its capabilities in adapting to local landscape and contracts on to the final minimum.

In recent years, researchers had proposed numerous optimization approaches to increase its convergence speed and accuracy. The simplex method approach was applied into particle swarm optimization method to solve the reactive power and voltage control problem. Moreover, (Tilahun & Ong, 2012) used the similar concept as described by (Nelder & Mead, 1964) in FA. However, their focus is on modifying random movement direction and determining the best direction in which the brightness increases.

In this perspective, the simplex method concept is adapted in FA in this paper. However, the simplex method is modified for more efficient approach, by adding center point as reference for searching other related points.

In this paper, the proposed DSFA had implemented the modified version of simplex method into FA. The DSFA uses four types of operations to replace a new point. The four operations are: *reflection*, *expansion*, *compression* and *contraction*. DSFA placed the points in *n*-dimensional space and created a hyperplane points. Later, they are calculated using the four operations to obtain the best optimal point, pre-optimal point, worst point, center point, reflection point, expansion point, compression point and contraction point. DSFA is able to fit itself to local landscape by elongating down long inclined planes, changing direction when it encounters a valley at an angle, and contracting around a minimum. This approach is easy to implement, has wider point searching area and converged fast.

**A. Dual Search Method**

Let  $x_L$  be lowest location for firefly, and  $x_C$  as the center location between best location and bottom location of firefly. Then,

Reflection Operation:  $x_R = x_C + \delta(x_C - x_L)$ ,  $x_R$  is reflection point, reflection coefficient,  $\delta$  by default is 1.

Expansion Operation:  $x_E = x_C + \phi(x_R - x_C)$ ,  $x_E$  is expansion point, expansion coefficient,  $\phi$  by default is 2.

Compression Operation:  $x_P = x_C + \varphi(x_L - x_C)$ ,  $x_P$  is compression point, compression coefficient,  $\varphi$  by default is 0.5.

Contraction Operation:  $x_T = x_C - \varphi(x_L - x_C)$ ,  $x_T$  is contraction point.

The steps of Dual Search method and operations are described as following:

**Step 1:** Calculate all point objective function values, locate the best point,  $x_G$ , and bottom point,  $x_B$ , hence, the objective for these points can expressed as  $f(x_G)$  and  $f(x_B)$ . Calculate the center point via

$$x_C = (x_G + x_B) / 2.$$

**Step 2:** Find the location of the lowest value firefly,  $x_L$ , with objective function value  $f(x_L)$ . Use  $x_L$  to find the reflection point  $x_R$ .

**Step 3:** If  $f(x_R) > f(x_G)$ , mean the reflection direction is correct, proceed calculation to get expansion point  $x_E$ . If  $f(x_E) > f(x_G)$ , use  $x_E$  to replace  $x_L$ , otherwise, use  $x_R$  replace  $x_L$ .

**Step 4:** If  $f(x_R) < f(x_L)$ , mean the reflection direction incorrect, proceed calculation to get compression point  $x_P$ . If  $f(x_P) > f(x_L)$ , use  $x_P$  to replace  $x_L$ .

**Step 5:** If  $f(x_L) < f(x_R) < f(x_G)$ , proceed calculation to get contraction point  $x_T$ . If  $f(x_T) > f(x_L)$ , use  $x_T$  to replace  $x_L$ , else, use  $x_R$  to replace  $x_L$ .

**B. The Steps of DSFA**

From FA described previously, here are the steps involved in DSFA and it is described as follows:

**Step 1:** Initialize FA parameters such as, initial firefly population,  $m$ , randomization parameter,  $\alpha$ , initial attractiveness,  $\beta_0$ , minimum value of attractiveness,  $\beta_{min}$ , light absorption coefficient,  $\gamma$ , and maximum iteration.

**Step 2:** Predetermine the location of  $m$  fireflies at  $X_i (i = 1, 2, \dots, m)$ , and Initial light intensity for each firefly,  $I_0$ .

**Step 3:** Calculate each firefly light intensity,  $I$ , and firefly attractiveness,  $\beta$ , using Equation (2.1) and Equation (2.3).

**Step 4:** Move firefly- $i$  to more attractive firefly- $j$  using Equation (2.4).

**Step 5:** Updating firefly location and light intensity according to Dual Search method as described in Section 3.1.

**Step 6:** Determine whether the termination criteria are satisfied, if is fulfill move to Step 7, else move to Step 3 and continue the flow again.

**Step 7:** Display optimize result.

**IV. EMPIRICAL SIMULATION RESULTS**

In this paper, the proposed method was applied to test 6 different standard functions, 2 functions with constrained and surface roughness data (Asilturk et al., 2016), then compared the results between FA and DSFA for their convergence rate and precision. The simulations are run via MATLAB 2012a. The default reference values for both algorithm are: initial number of firefly  $N = 20$ , randomization parameter  $\alpha = 0.2$ , initial attractiveness  $\beta_0 = 1$ , minimum value of attractiveness  $\beta_{min} = 0.2$ , light absorption coefficient  $\gamma = 1$ , and maximum iteration is set as 200.

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**C. Test on Standard Functions**

For standard functions test, each function is executed for 20 times and the average value for simulation results are obtained. Table II shows the test functions and its parameter value, while Table III shows the comparison results for FA and DSFA for their best objective value, worst objective value, average objective value and standard deviations. Furthermore, the comparison of convergence performances for each function was included in Figure 2 to Figure 10.

**Table II. The test function and its parameter value**

Function	Optimal Range	Dimension	Minimum Point
$f_{Sphere}(x) = \sum_{i=1}^N x_i^2$	[-100,100]	30	0
$f_{Absolute}(x) = \max \{ x_i \}$	[-10,10]	30	0
$f_{Mix}(x) = -20e^{\left(-0.2\sqrt{\frac{1}{N}\sum_{i=1}^N x_i^2}\right)} - e^{\left(\frac{1}{N}\sum_{i=1}^N \cos(2\pi x_i)\right)} + 20 + e$	[-32,32]	30	0
$f_{Rosenbrock}(x) = \sum_{i=1}^{N-1} \left[100(x_i^2 - x_{i+1})^2 + (x_i - 1)^2\right]$	[-5,10]	4	0
$f_{Beale}(x) = (1.5 - x_1 + x_1 x_2)^2 + (2.25 - x_1 + x_1 x_2^2)^2 + (2.625 - x_1 + x_1 x_2^3)^2$	[-4.5,4.5]	2	0
$f_{Colville}(x) = 100(x_1^2 - x_2^2) + (x_1 - 1)^2 + (x_3 - 1)^2 + 90(x_3^2 - x_4)^2 + 10.1[(x_2 - 1)^2 + (x_4 - 1)^2] + 19.8(x_2 - 1)(x_4 - 1)$	[-10,10]	4	0
$f_{Rosenbrock}(x) = (1 - x_1)^2 + 100(x_2 - x_1^2)^2$ , Subjected to: $x_1^2 + x_2^2 < 2$	[-1.5,1.5]	2	0
$f_{Mishra}(x) = \sin(x_2)e^{(1-\cos(x_1))^2} + \cos(x_1)e^{(1-\sin(x_2))^2} + (x_1 - x_2)^2$ , Subjected to: $(x_1 + 5)^2 + (x_2 + 5)^2 < 25$	[-10,0]	2	-106.764536
$f_{Surface}(x) = -2.957123 + 0.014958x_1 + 67.816843x_2 + 0.149618x_3 - 10.338616x_4 - 0.066869x_1x_2 - 0.001995x_1x_3 + 0.001689x_1x_4 - 16.576147x_2x_3 - 26.576147x_2x_4 + 6.147745x_3x_4 - 0.000004x_1^2 + 39.874074x_2^2 - 1.145833x_3^2 + 4.023958x_4^2$	$318 \leq x_1 \leq 636$ $0.001 \leq x_2 \leq 0.0025$ $0.005 \leq x_3 \leq 0.009$ $0.004 \leq x_4 \leq 0.012$	4	0.81

From Table III, clearly in comparison of DSFA and FA, overall the performances of DSFA outperformed the FA in all results. From standard deviation results, DSFA could provide more precise and stable results with low deviation from each simulation run. This means that DSFA provides solutions that are close to targeted result.

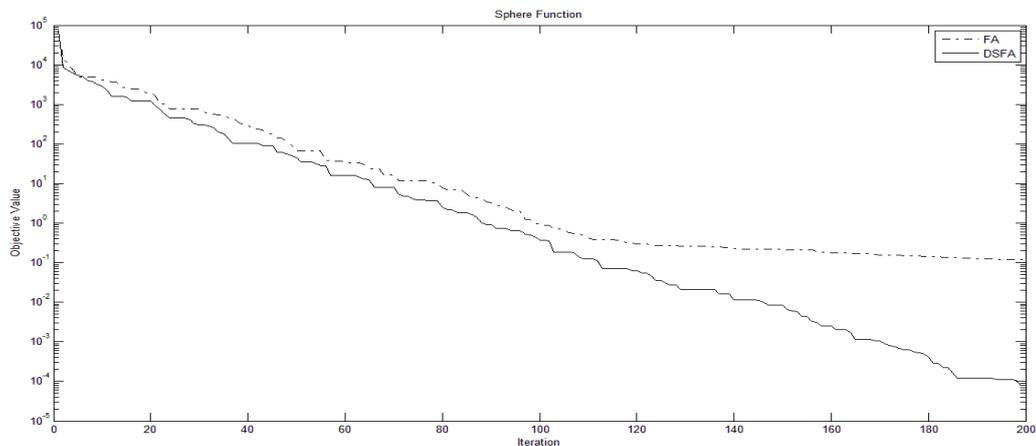
For  $f_{Sphere}$  function, the DSFA average objective value performed 99.9% more accurate than FA. Furthermore, we observed that DSFA achieved best objective value and standard deviation at precision of  $1 \times 10^{-5}$  instead of  $1 \times 10^{-2}$  for FA. Then for function  $f_{Absolute}$ , the DSFA accuracy is 89.9% higher than FA in average objective value. For function  $f_{mix}$ , the DSFA showed 99.5% higher accuracy compared to FA. In addition, we observed that DSFA achieved best objective value at precision of  $1 \times 10^{-3}$  instead of  $1 \times 10^{-1}$  for FA for both function  $f_{Absolute}$  and  $f_{mix}$ .

Then for  $f_{Rosenbrock}$ , DSFA is 99.9% accurate than FA in average objective value. Here, it is observed that DSFA achieved stable standard deviation at precision of  $1 \times 10^{-9}$  instead of  $1 \times 10^{-3}$  for FA. Next, for function  $f_{Beale}$ , the performance of DSFA is outperformed FA for 77% in average objective value. Finally, function  $f_{Colville}$  showed 99.9% higher accuracy in compare to FA in average objective value. Finally, function  $f_{Colville}$  showed 99.9% higher accuracy in compare to FA in average objective value. Additionally, the DSFA shows good stability

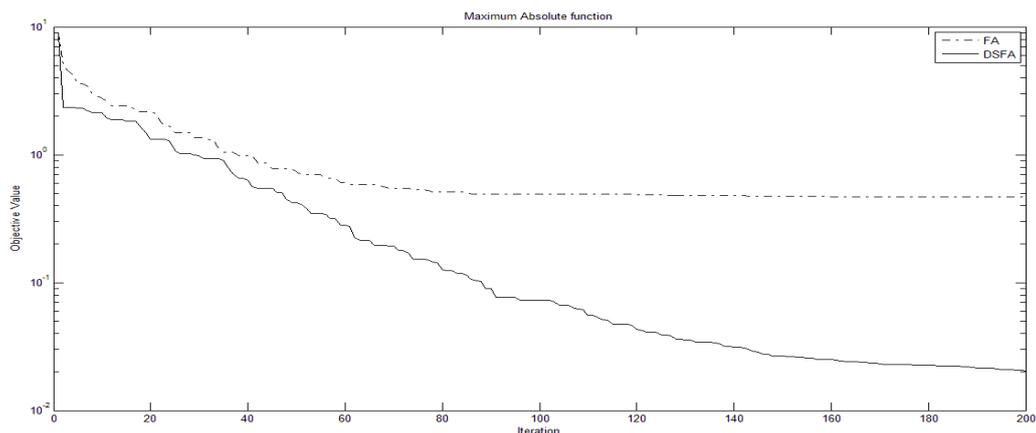
in standard deviation at precision of  $1 \times 10^{-5}$  as compared to FA. From Figure 2 to Figure 7, one can see the convergence curve for DSFA is faster than FA, and DSFA had more accurate objective value compared to FA.

**Table III. Comparison of experiments results of FA and DSFA**

Function	Algorithm	Best Obj	Worse Obj	Average Obj	Standard Deviation
$f_{\text{Sphere}}(X)$	FA	0.02923353	0.11672873	0.066818373	0.02558059
	DSFA	3.9399E-05	8.7487E-05	6.26632E-05	1.19337E-05
$f_{\text{Absolute}}(X)$	FA	0.18717447	0.88369369	0.524745211	0.182828406
	DSFA	0.00907296	0.12668163	0.052800118	0.038544014
$f_{\text{Mix}}(X)$	FA	0.17940552	0.98188577	0.430329017	0.214496061
	DSFA	0.00155959	0.0024315	0.001907527	0.000189965
$f_{\text{Rosenbrock}}(X)$	FA	1.0999E-09	0.0090789	0.000527169	0.002027143
	DSFA	4.3449E-11	4.1843E-09	1.22475E-09	1.19994E-09
$f_{\text{Beale}}(X)$	FA	1.015E-11	7.4558E-10	3.14259E-10	2.01596E-10
	DSFA	3.5406E-12	1.7447E-10	7.11302E-11	4.51672E-11
$f_{\text{Corville}}(X)$	FA	0.00022436	5.17302563	1.763967864	1.93204155
	DSFA	2.0392E-07	4.7769E-05	7.57066E-06	1.16928E-05
$f_{\text{Rosenbrock\_C}}(X)$	FA	5.3051E-10	0.05786845	0.010039409	0.017224815
	DSFA	8.942E-12	6.5541E-10	1.0011E-10	1.46772E-10
$f_{\text{Mishra\_C}}(X)$	FA	-106.76454	-36.929558	-89.81173885	20.97475972
	DSFA	-106.76454	-87.310883	-101.1076048	8.299690684
$f_{\text{Surface}}(X)$	FA	3.2716E-08	0.87587797	0.047379289	0.19566454
	DSFA	3.2655E-09	1.8899E-07	7.48563E-08	6.05788E-08



**Figure 1. Convergence curve of sphere function**



**Figure 2. Convergence curve of the sum of maximum absolute function**

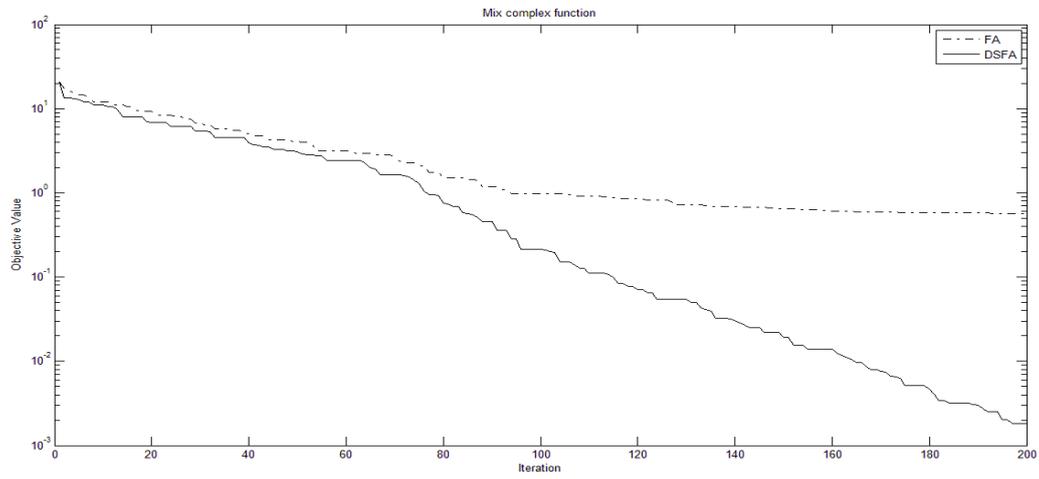


Figure 3. Convergence curve of mix complex function

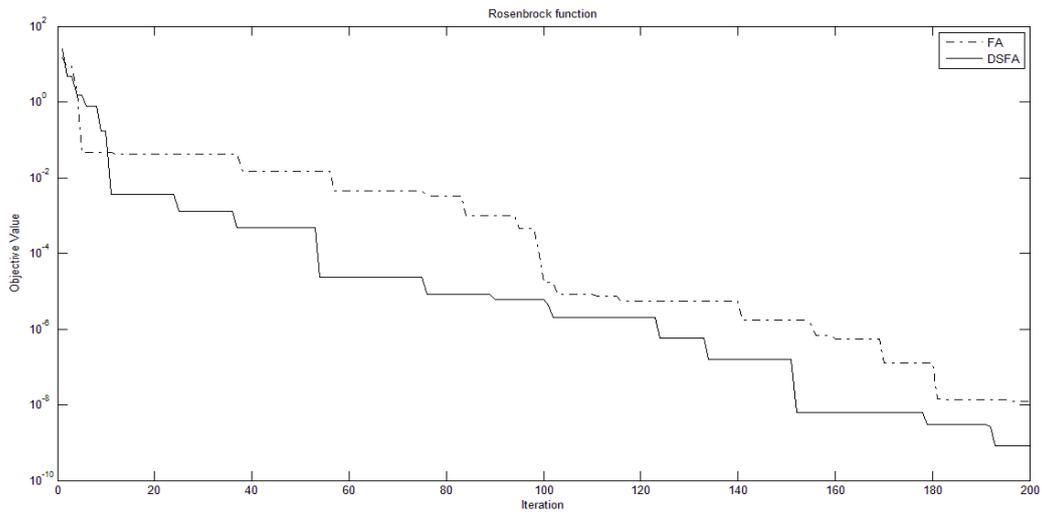


Figure 4. Convergence curve of Rosenbrock function

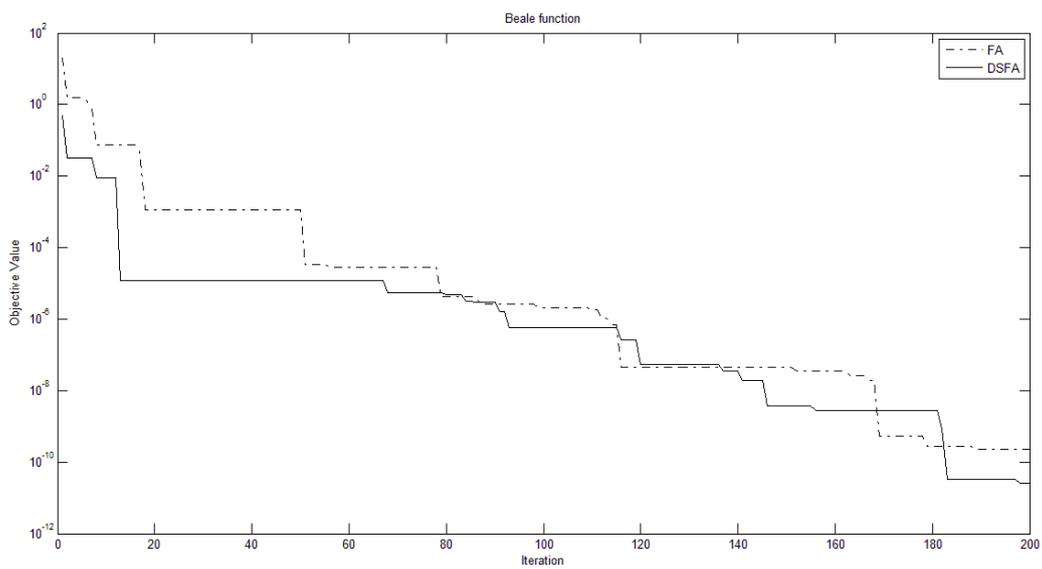
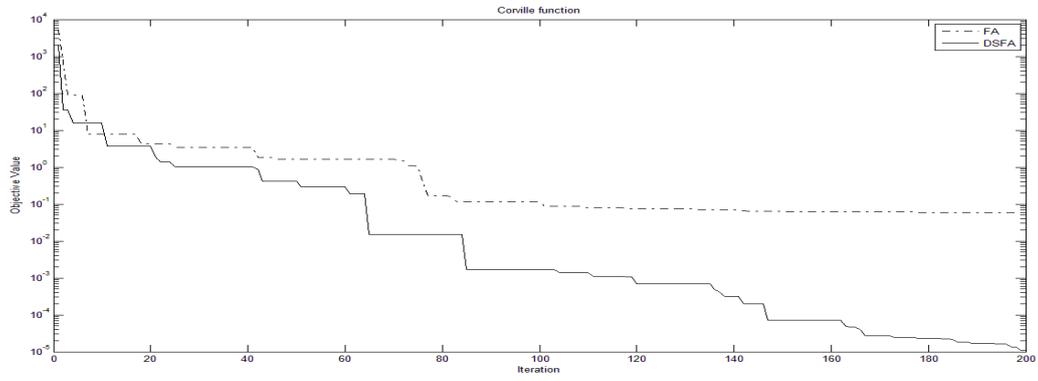
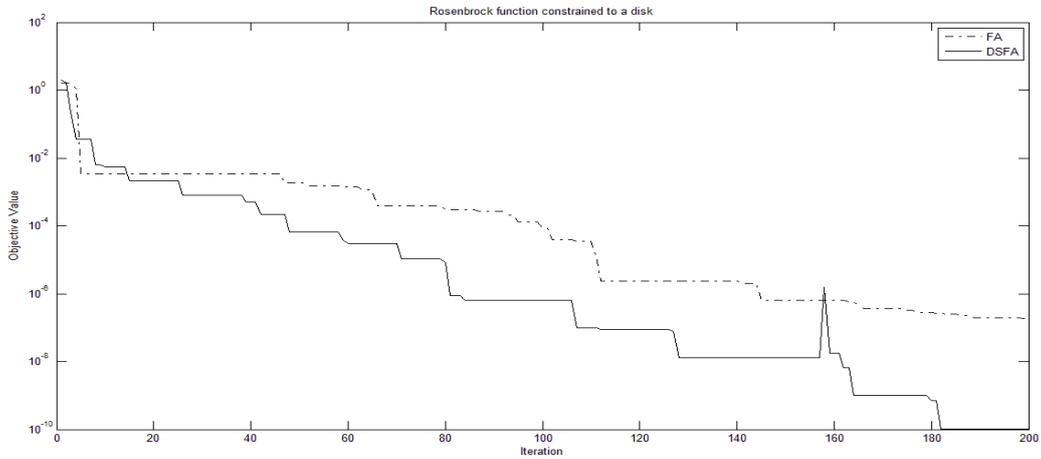


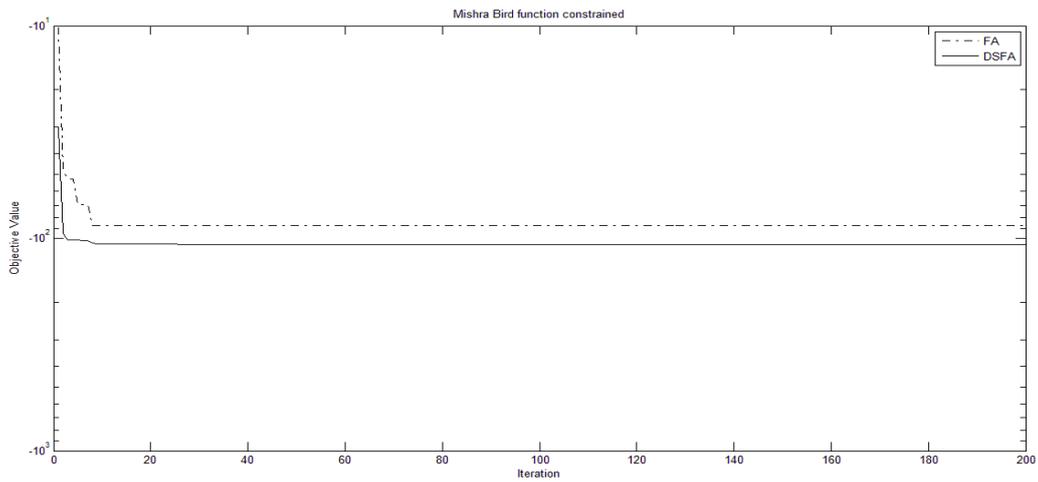
Figure 5. Convergence curve of Beale function



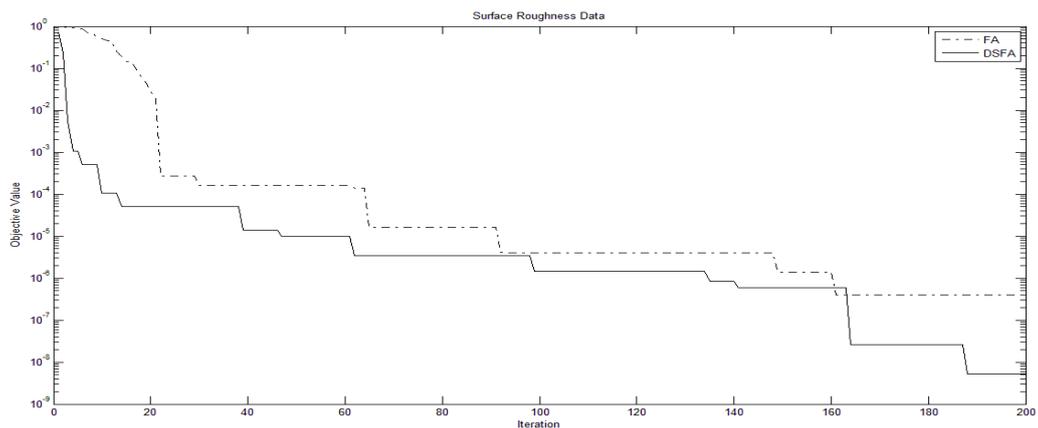
**Figure 6. Convergence curve of Corville function**



**Figure 7. Convergence curve of Rosenbrock function with constraint**



**Figure 8. Convergence curve of Mishra function with constraint**



**Figure 9. Convergence curve of surface roughness data**

#### D. Test on Function With Constraint

From Table III, the comparison of results for DSFA and FA, clearly showed the overall performance of DSFA, which outperformed FA for function  $f_{Rosenbrock\_C}$  and  $f_{Mishra\_C}$ . For  $f_{Rosenbrock\_C}$  function, DSFA could achieve precision of  $1 \times 10^{-10}$  in average objective value and standard deviation, yet FA only achieved  $1 \times 10^{-2}$  for both results.

For  $f_{Mishra\_C}$  function, the DSFA and FA each obtained average objective values of 5.6% and 15.9% deviated from targeted value respectively. Furthermore, it is clearly seen that DSFA achieved 60.4% reduction in standard deviation as compared to FA. Figure 8 and Figure 9 displayed the convergence curve of both constraint function for this subsection. Clearly, DSFA converges faster and provides better accuracy in objective value. Results from both constraint function shows that, DSFA outperform FA and can provide high accuracy and stability in optimization result.

#### E. Test on Surface Roughness Data

For this test, the data are obtained from the study of (Asiltürk, Neşeli, & Ince, 2016). The data consists of four dimensions: rotational speed, feed rate, depth of cut, and tool tip radius. Here, the output values are surface roughness value in  $\mu m$ . Further details on the data can be found in (Asiltürk et al., 2016). This test is labeled with function  $f_{Surface}$ .

From the result in Table III, the average objective value for both DSFA gain 99.9% higher accuracy than FA. Furthermore, the standard deviation value for DSFA achieved precision of  $1 \times 10^{-8}$  instead of  $1 \times 10^{-1}$  for FA. This shows that DSFA is stable in performance and had high precision value. Finally, from Figure 10, it is shown that DSFA accelerated in convergence in early and last stages of iteration. Hence, this boosts DSFA performances to obtain lower objective function than FA, which achieved higher accuracy in result.

#### V. CONCLUSION

In this paper, the principle and algorithm description of FA and developmental history and research status of the algorithm were introduced in detail. Also, the developmental prospect of FA was presented.

Here, a new dual search firefly algorithm (DSFA) based on modified simplex method is proposed to solved the problem that conventional FA had, such as, slow convergence speed, easily trapped in local optimum and has low precision result. In DSFA, the search strategy improves the diversity of the individuals by using reflection, expansion, compression and contraction points to improve the diversity of the fireflies, and allowing them to effectively avoid trapping in the local optimum, to improve the accuracy of the algorithm. Moreover, from results of standard functions, functions with constraint and surface roughness data of the simulation experiments, showed that the effectiveness, validity and feasibility of DSFA, are better than FA in the ability to escape local optimal trap and gives higher solution accuracy with robustness.

#### REFERENCES

- [1]. **Abdullah, A., Deris, S., Mohamad, M. S., & Hashim, S. Z. M. (2012).** A new hybrid firefly algorithm for complex and nonlinear problem. In *Advances in Intelligent and Soft Computing* (Vol. 151 AISC, pp. 673–680). [https://doi.org/10.1007/978-3-642-28765-7\\_81](https://doi.org/10.1007/978-3-642-28765-7_81)
- [2]. **Akhoondzadeh, M. (2015).** Firefly Algorithm in detection of TEC seismo-ionospheric anomalies. *Advances in Space Research*, 56(1), 10–18. <https://doi.org/10.1016/j.asr.2015.03.025>
- [3]. **Anon. (2017).** Test Functions for Optimization. Retrieved March 17, 2017, from [https://en.wikipedia.org/wiki/Test\\_functions\\_for\\_optimization](https://en.wikipedia.org/wiki/Test_functions_for_optimization)
- [4]. **Asiltürk, I., Neşeli, S., & Ince, M. A. (2016).** Optimisation of parameters affecting surface roughness of Co28Cr6Mo medical material during CNC lathe machining by using the Taguchi and RSM methods. *Measurement: Journal of the International Measurement Confederation*, 78(November 2015), 120–128. <https://doi.org/10.1016/j.measurement.2015.09.052>
- [5]. **Chandrasekaran, K., & Simon, S. P. (2012).** Network and reliability constrained unit commitment problem using binary real coded firefly algorithm. *International Journal of Electrical Power and Energy Systems*, 43(1), 921–932. <https://doi.org/10.1016/j.ijepes.2012.06.004>
- [6]. **Colorni, A., Dorigo, M., & Maniezzo, V. (1991).** Distributed Optimization by Ant Colonies. *Proceedings of the First European Conference on Artificial Life*, 142(or D), 134–142. [https://doi.org/10.1016/S0303-2647\(97\)01708-5](https://doi.org/10.1016/S0303-2647(97)01708-5)
- [7]. **Dos Santos Coelho, L., & Mariani, V. C. (2013).** Improved firefly algorithm approach applied to chiller loading for energy conservation. *Energy and Buildings*, 59, 273–278. <https://doi.org/10.1016/j.enbuild.2012.11.030>
- [8]. **Faro, A., & Giordano, D. (2016).** Algorithms to find shortest and alternative paths in free flow and congested traffic regimes. *Transportation Research Part C: Emerging Technologies*, 73, 24–28. <https://doi.org/10.1016/j.trc.2016.09.009>
- [9]. **Gandomi, A. H., Yang, X. S., & Alavi, A. H. (2011).** Mixed variable structural optimization using Firefly Algorithm. *Computers and Structures*, 89(23–24), 2325–2336. <https://doi.org/10.1016/j.compstruc.2011.08.002>
- [10]. **Gandomi, A. H., Yang, X. S., Talatahari, S., & Alavi, A. H. (2013).** Firefly algorithm with chaos. *Communications in Nonlinear Science and Numerical Simulation*, 18(1), 89–98. <https://doi.org/10.1016/j.cnsns.2012.06.009>
- [11]. **Gnana Sundari, M., Rajaram, M., & Balaraman, S. (2016).** Application of improved firefly algorithm for programmed PWM in multilevel inverter with adjustable DC sources. *Applied Soft Computing Journal*, 41, 169–179. <https://doi.org/10.1016/j.asoc.2015.12.036>

- [12]. **Gokhale, S. S., & Kale, V. S. (2016).** An application of a tent map initiated Chaotic Firefly algorithm for optimal overcurrent relay coordination. *International Journal of Electrical Power and Energy Systems*, 78, 336–342. <https://doi.org/10.1016/j.ijepes.2015.11.087>
- [13]. **He, Q., Hu, X., Ren, H., & Zhang, H. (2015).** A novel artificial fish swarm algorithm for solving large-scale reliability-redundancy application problem. *ISA Transactions*, 59, 105–113. <https://doi.org/10.1016/j.isatra.2015.09.015>
- [14]. **Holland. (1975).** *Adaptation in Natural and Artificial System.* The University of Michigan Press.
- [15]. **Horng, M. H., & Liou, R. J. (2011).** Multilevel minimum cross entropy threshold selection based on the firefly algorithm. *Expert Systems with Applications*, 38(12), 14805–14811. <https://doi.org/10.1016/j.eswa.2011.05.069>
- [16]. **Ji, D.-M., Sun, J.-Q., Dui, Y., & Ren, J.-X. (2017).** The optimization of the start-up scheduling for a 320 MW steam turbine. *Energy*, 125, 345–355. <https://doi.org/10.1016/j.energy.2017.02.139>
- [17]. **Kazem, A., Sharifi, E., Hussain, F. K., Saberi, M., & Hussain, O. K. (2013).** Support vector regression with chaos-based firefly algorithm for stock market price forecasting. *Applied Soft Computing*, 13(2), 947–958. <https://doi.org/10.1016/j.asoc.2012.09.024>
- [18]. **Kennedy, J., & Eberhart, R. (1995).** Particle swarm optimization. *Neural Networks*, 1995. *Proceedings., IEEE International Conference On*, 4, 1942–1948 vol.4. <https://doi.org/10.1109/ICNN.1995.488968>
- [19]. **Kougianos, E., & Mohanty, S. P. (2015).** A nature-inspired firefly algorithm based approach for nanoscale leakage optimal RTL structure. *Integration, the VLSI Journal*, 51, 46–60. <https://doi.org/10.1016/j.vlsi.2015.05.004>
- [20]. **Laskari, E. C., Parsopoulos, K. E., & Vrahatis, M. N. (2002).** Particle swarm optimization for integer programming. *Proceedings of the 2002 Congress on Evolutionary Computation. CEC'02 (Cat. No.02TH8600)*, 2, 1582–1587. <https://doi.org/10.1109/CEC.2002.1004478>
- [21]. **Long, N. C., Meesad, P., & Unger, H. (2015).** A highly accurate firefly based algorithm for heart disease prediction. *Expert Systems with Applications*, 42(21), 8221–8231. <https://doi.org/10.1016/j.eswa.2015.06.024>
- [22]. **Nelder, J. a., & Mead, R. (1964).** A simplex method for function minimization. *Computer Journal*, 7, 308–313. <https://doi.org/10.1093/comjnl/7.4.308>
- [23]. **Rahmani, A., & MirHassani, S. A. (2014).** A hybrid Firefly-Genetic Algorithm for the capacitated facility location problem. *Information Sciences*, 283, 70–78. <https://doi.org/10.1016/j.ins.2014.06.002>
- [24]. **Rajan, A., & Malakar, T. (2015).** Optimal reactive power dispatch using hybrid Nelder-Mead simplex based firefly algorithm. *International Journal of Electrical Power and Energy Systems*, 66, 9–24. <https://doi.org/10.1016/j.ijepes.2014.10.041>
- [25]. **S.M. Farahani, A. Abshouri, B. Nasiri, M. Meybodi. (2012).** Some Hybrid Models to Improve Firefly Algorithm Performance. *International Journal of Artificial Intelligence*, 8(12), 97–117.
- [26]. **Sayadi, M. K., Ramezani, R., & G. N., N. (2010).** A discrete firefly meta-heuristic with local search for makespan minimization in permutation flow shop scheduling problems. *International Journal of Industrial Engineering Computations*, 1(1), 1–10. <https://doi.org/10.5267/j.ijiec.2010.01.001>
- [27]. **Senthilnath, J., Omkar, S. N., & Mani, V. (2011).** Clustering using firefly algorithm: Performance study. *Swarm and Evolutionary Computation*, 1(3), 164–171. <https://doi.org/10.1016/j.swevo.2011.06.003>
- [28]. **Srivatsava, P. R., Mallikarjun, B., & Yang, X. S. (2013).** Optimal test sequence generation using firefly algorithm. *Swarm and Evolutionary Computation*, 8, 44–53. <https://doi.org/10.1016/j.swevo.2012.08.003>
- [29]. **Tilahun, S. L., & Ong, H. C. (2012).** Modified firefly algorithm. *Journal of Applied Mathematics*, 2012. <https://doi.org/10.1155/2012/467631>
- [30]. **Udaiyakumar, K. C., & Chandrasekaran, M. (2014).** Application of firefly algorithm in job shop scheduling problem for minimization of Makespan. *Procedia Engineering*, 97, 1798–1807. <https://doi.org/10.1016/j.proeng.2014.12.333>
- [31]. **Verma, S., Saha, S., & Mukherjee, V. (2016).** A novel symbiotic organisms search algorithm for congestion management in deregulated environment. *Journal of Experimental and Theoretical Artificial Intelligence*, 19(3), 1–12. <https://doi.org/10.1080/0952813X.2015.1132269>
- [32]. **Verwer, S., Zhang, Y., & Ye, Q. C. (2015).** Auction optimization using regression trees and linear models as integer programs. *Artificial Intelligence*, 1, 37. <https://doi.org/10.1016/j.artint.2015.05.004>
- [33]. **Yang, X.-S. (2009).** Firefly Algorithms for Multimodal Optimization. *Proceedings of the 5th International Conference on Stochastic Algorithms: Foundations and Applications*, 169–178. [https://doi.org/10.1007/978-3-642-04944-6\\_14](https://doi.org/10.1007/978-3-642-04944-6_14)
- [34]. **Yang, X.-S. (2010).** Firefly Algorithm, Levy Flights and Global Optimization. *Research and Development in Intelligent Systems XXVI: Incorporating Applications and Innovations in Intelligent Systems XVII*, 1–10. <https://doi.org/10.1007/978-1-84882-983-1-15>
- [35]. **Yang, X.-S. (2014).** Nature-Inspired Optimization Algorithms. *Nature-Inspired Optimization Algorithms*. <https://doi.org/10.1016/B978-0-12-416743-8.00001-4>
- [36]. **Yang, X. S., & Deb, S. (2010).** Eagle strategy using Levy walk and firefly algorithms for stochastic optimization. In *Studies in Computational Intelligence (Vol. 284, pp. 101–111)*. [https://doi.org/10.1007/978-3-642-12538-6\\_9](https://doi.org/10.1007/978-3-642-12538-6_9)
- [37]. **Yang, X. S., Hosseini, S. S. S., & Gandomi, A. H. (2012).** Firefly Algorithm for solving non-convex economic dispatch problems with valve loading effect. *Applied Soft Computing Journal*, 12(3), 1180–1186. <https://doi.org/10.1016/j.asoc.2011.09.017>

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