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Value at risk (VAR)
Non-parametric methodologies
An approach to the study and application

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ABSTRACT

VAR (Value at Risk), is the maximum probable loss of a portfolio for a given level of confidence, within a specified time horizon. In Instruction No. 4/2008, Banco de Portugal defines Value-at-risk to be "the maximum expected loss that is not exceeded with a specific probability (confidence level) and for a certain period of time (detention period) ".

There are several methodologies for estimating VAR, but they are divided into two large groups: non-parametric (Historical simulations and Monte Carlo simulations) and parametric, based on variance and covariancy.

We will only look at non-parametric methodologies:

- Historical Simulations that are based on the assumption that the future change in the prices of the relevant assets in the portfolio, is distributed in the same way as in the past;
- Monte Carlo simulations whose difference to historical simulations lies in the way of obtaining the simulated scenarios, which constitute a sample generated in a (pseudo) random way taking into account a given distribution.

KEYWORDS: *VAR*; historical simulations; Monte Carlo simulations, finance portfolio. JEL: classification G32

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I. INTRODUCTION

VAR (Value at risk) is the maximum probable loss of a portfolio for a given level of confidence, within a specified time horizon. In this way, the VAR measures the most pessimistic estimated potential losses, which are calculated taking into account a certain level of confidence, for a given period of time and under normal market conditions.

The VAR methodologies consider the risk and the degree of uncertainty in relation to future net returns. However, they base the projection of future price changes on the distribution of historical market values for those same prices.

A VAR of 100 € (one hundred euros) means that, on average, taking into account a 5% risk level, only in 1 day in 20, you can expect that there will be a loss greater than 100 due to price fluctuations from the market.

Thus, VAR methodologies are based on normal market movements , or that is, it presupposes the absence of major financial crises.

Markets where there is significant volatility undermine the effect of correlation between prices and liquidity risk (difficulty in liquidating positions) and may distort the value obtained by the VAR. It is a measure that allows quantifying the potential loss of a portfolio associated with a statistically defined level of confidence. Its validity is marked out in time.

The methodologies to estimate the VAR can be several, namely, techniques based on variance and covariance (called methodologies or parametric models), in which the following specific models are highlighted.

- Risk Metrics - JP Morgan
- Raroc 2020 - Bankers Trust
- Prime Rise - Credit Suisse First Boston
- Risk Dollars - Chase M. Bank
- "Group of the thirties of 1993-"Derivatives: Practices and Principles"
- Historical simulations and Monte-Carlo simulations (scenarios) by non-parametric methodologies and which we will address below).

We can gather the main differences between the methodologies in the following table:

Table 1: Difference on methodologies

	Non-parametric		Parametric
Functions	Historical simulation	Monte Carlo	Variance - Covariance
Distribution definition	The distribution of historical data is calculated	Values are generated	Standard deviation and correlation are estimated
Calculation of portfolio distribution	The portfolio values are simulated		The standard deviation of the portfolio is calculated, assuming a normal distribution.
Obtaining the VAR	Loss values are ordered and all that exceed (1-p) probability are selected		

Source: authors elaboration

Annex-1 shows comparative tables with the advantages / disadvantages of each methodology.

II. VAR (Historical Simulation – Non - Parametric Methodology)

2.1 Principles

This methodology is based on the hypothesis of the stationarity of the data, which assumes that history and the best possible stimulation of the future.

Therefore, the principle is to estimate the distribution of changes in future prices from historical prices and apply these variations to the current portfolio to determine the VAR. This method is said to be non-parametric because, unlike other approaches, eg. Risk Metrics, calculating the VAR does not imply having an estimate of the parameters of a theoretical distribution, such as the normal one.

It should be noted that it is the entire portfolio that is taken into account and not each individual asset. The VAR of a portfolio is not the sum of the individual VAR's for each instrument but the VAR for the entire portfolio, which allowstake into account the effect of the correlations.

One of the institutions that uses this method is JP Morgan Chase, which uses a 100-day historical series for a 1-day time horizon. (in this situation the VAR is called DEAR, as it applies to daily results)

2.2 Examples

In the example presented, we will only have 20 daily observations of two titles, in order not to overload the calculations. The steps to calculate the VAR according to the Historical simulation are as follows (see table 2):

- The observations concern the prices of two securities: 5 units of title 1 and 10 units of title 2;
- The changes for each period in relation to the previous day are calculated in relative value (rate of change) using the formula: (pt - pt-1) / pt-1, where pt is the price of the day and pt-1 the price of the day previous. So, for example, for the first of the days before the current one we have: (205-200) / 200 = 2.5% for title 1 and (210-220) / 210 = 4.5% for title 2;
- Each security is valued by applying the variation of the day from its value in the current period (t0). The calculation consists of subjecting the current portfolio to changes in the past. The formula: p0 (1 + variation t), where p0 is the current value and variation is the variation of the day. For day -19, we have 215 x (1 + 2.5%) = 220 and 160 x (1-4.55%) = 153 for title 2. For day 18, we have 215 x (1 + 2.44%) = 220 for title 1 and 160 x (1 + 9.5%) = 175 for title 2;
- The total value of the portfolio is calculated according to the number of securities held. So for the first day, we have: 220 x 5 + 153 x 10 = 2629.
- Daily losses are calculated in relation to the current value of the portfolio (5 x 215 + 10 x 160 = 2675). Losses are recorded with a positive value and gains with a negative value (as a matter of ordering the values).
- Losses are classified in ascending order and the reading is made on the column of the amount of the VAR according to the desired confidence level.
- The Probability column is calculated considering that each observation represents 5% (twenty observations) cumulatively.

Table 2: calculation of the VAR Historical simulation

Day	Historic		Variations		Portfolio evaluation		Total	Current portfolio value	Losses	Ordered losses	Probability
	Security 1	Security 2	Security 1	Security 2	Security 1	Security 2					
-20	200	220									
-19	205	210	2.50%	-4.55%	220	153	2629	2675	46	-245	0.05
-18	210	230	2.44%	9.52%	220	175	2854	2675	-179	-179	0.1
-17	195	250	-7.14%	8.70%	200	174	2737	2675	-62	-142	0.15
-16	205	230	5.13%	8.00%	226	147	2602	2675	73	-134	0.2
-15	190	210	-7.32%	8.70%	199	146	2457	2675	218	-125	0.25
-14	190	180	0.00%	-14.29%	215	137	2446	2675	229	-104	0.3
-13	220	150	15.79%	-16.67%	249	133	2578	2675	97	-87	0.35
-12	180	180	-18.18%	20.00%	175	192	2800	2675	-125	-62	0.4
-11	215	160	-18.44%	-11.11%	257	142	2706	2675	-31	-31	0.45
-10	200	170	-6.98%	6.25%	200	170	2700	2675	-25	-25	0.5
-9	190	190	-5.00%	11.76%	204	179	2809	2675	-134	9	0.55
-8	230	180	21.05%	-5.26%	260	152	2817	2675	-142	23	0.6
-7	190	200	-17.39%	11.11%	178	178	2666	2675	9	46	0.65
-6	205	220	7.89%	10.00%	232	176	2920	2675	-245	73	0.7
-5	190	205	-7.32%	-6.82%	199	149	2487	2675	188	97	0.75
-4	195	180	2.63%	-12.20%	221	140	2508	2675	167	167	0.8
-3	230	170	17.95%	-5.56%	254	151	2779	2675	-104	188	0.85
-2	205	180	-10.87%	5.88%	192	169	2652	2675	23	207	0.9
-1	230	175	12.20%	-2.78%	241	156	2762	2675	-87	218	0.95
0	215	160	-6.52%	-8.57%	201	146	2468	2675	207	229	1

Source: authors elaboration

Thus, it is likely that the loss will not exceed more than the amount indicated for the desired confidence level, that is:

- In 70% of the cases: 73 (as mentioned in the positive represent losses);
- In 90% of the cases: 207;

- In 95% of cases: 218

Another example, in table 3 is presented below.

Consider a portfolio consisting of 6 securities 1 and 8 securities 2 and the stock market behavior in the last 8 days was as follows:

Table 3 - stock market behavior (historical)

Historic		
Day	**Security 1**	**Security 2**
-8	50	65
-7	70	66
-6	75	58
-5	60	62
-4	55	63
-3	78	60
-2	90	55
-1	95	56
0	80	55

Source: authors elaboration

It is now important to determine the VAR Historical simulation - non-parametric model for the following confidence levels (see table 4):

a) 62.5%
b) 75%

Table 4: Solution

112x6 + 56x8 180 X 6 t 55 X 8

80 x (1 + 0.4), with 80 being

(70-50) / 50 current price

Day	Historic		Variations		Evaluation of the portfolio		Total	Portfolio value		Ordered losses	Accumulated %
	Security 1	Security 2	Security 1	Security 2	Security 1	Security 2		Current	Losses		
					6	8					
-8	50	65									
-7	70	66	0.4	0.015385	112	56	1119	920	-199	-199	0.125
-6	75	58	0.071429	-0.12121	86	48	901	920	19	-180	0.25
-5	60	62	-0.2	0.068966	64	59	854	920	66	-37	0.375
-4	55	63	-0.08333	0.016129	73	56	887	920	33	-35	0.5
-3	78	60	0.418182	-0.04762	113	52	1100	920	-180	10	0.625
-2	90	55	0.153846	-0.08333	92	50	957	920	-37	33	0.75
-1	95	56	0.055556	0.018182	84	56	955	920	-35	66	0.875
0	80	55	-0.15789	-0.01786	67	54	836	920	84	84	1

Source: authors elaboration

In view of the table, the VAR assumes the following values:

a) VAR (62.5%): 10

b) VAR: (75%): 33

2.3 Advantages and disadvantages of this methodology

This methodology is simple to apply and requires few computer resources. It has the advantage that complex calculations are not necessary.

The VAR is determined over the entire portfolio and not and the sum of the individual VARs (that is, in the example shown above, the calculated VAR falls on 5 units of securities 1 and 10 units of security 2). Therefore, it takes into account the effect of the correlations.

However, if an asset has no history, the VAR - Historical simulation methodology cannot be applied. In this case, it is necessary to use simulations and complex calculations. On the other hand, if perhaps a small number of observations have very different values (values that are very far from the average, whether positive or negative, called outsiders), the calculation is highly addictive, so strict data control is essential. In the event of the existence of these values, it is advisable to carefully analyze whether or not other criticisms aimed at this methodology should be included in the calculations and which weighs all observations (historical data) in the same way, including the oldest prices.

III. VAR (Monte Carlo simulation - Non-Parametric Methodology)

3.1 Principles

Most of the models previously presented are effective in determining the value at risk (VAR) of investments in instruments that have a linear or non-linear behavior due to convexity, such as bonds and similar. But as soon as an instrument's behavior suffers abrupt and unpredictable disruptions, these models may not be the most suitable. For example, the price of an option can vary in an unpredictable manner due to a small change in the price of the underlying asset.

The main difference between Monte Carlo simulation and historical simulation lies in the way of obtaining the simulated scenarios.

Given a probability distribution, Monte Carlo[5] simulations (scenarios) constitute a generated sample of random (pseudo) form taking into account the said distribution. It is a repetitive process, generating deterministic solutions to a given problem. Each solution corresponds to a set of deterministic values of the underlying variables.

The distribution is defined by a set of parameters, which are, among others: matrix of variances and covariance, speeds and reversion rates in relation to the average.

The Monte Carlo simulation addresses the problem with a sequence of points in time, allowing to simulate evolutionary trajectories, for which it is necessary to define a vector of temporal nodules, as well as having a random number generator.

In the following section, some examples are presented for a better understanding of the Monte Carlo simulation.

3.2 Examples

The methodology to be used is the Monte Carlo random simulation technique (name of the famous casino). It consists in determining, for example, the VAR of an option, through the generation of a series of prices of the underlying instrument, in the order of 5000 to 10000, and in the study of the behavior of the option.

It should be noted that the option and an instrument whereby the entity that acquires it has the right (but not the obligation) to buy or sell a certain asset, within a certain period of time, at a previously agreed price (denominated price of exercise). In addition, taking into account the type of right (buy or sell), it is possible to divide option contracts into two categories:

1. Call options, which give the holder the right to buying an asset;

2. Put options, which give the holder the right to sell an asset

To simulate the option value calculation, the methodology (using the Black-Scholes model) adopts the following hypotheses:

a) The methodology is based on the distribution of the normal law that tells us that the probability that a value is close to its average is high, instead of the probability that a value is far from its average. An instrument with a value of 100 is unlikely to reach a value of 10 or 1000 in the near future, for example in a year. Random runs take this law into account to generate realistic numbers.

b) What is sought is not the variation of the instrument in absolute value but in value relative, therefore its yield. A variation of 1 does not have the same meaning if the instrument is worth 100 or 2. In the first case, the gain is 1% and in the second, 50%. Therefore, it is the yield, and not the share price, that follows a normal distribution (lognormal distribution).

The data of the example (succinct because only with 10 observations and without transaction expenses and specific exercise premium) - see table 5, are as follows:

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- Original share price (ie the underlying asset of the option): 100
- Standard deviation of the share price: 20%
- Average rate of return (equal to the discount rate) for this share category = 10%
- Time frame for analysis (time horizon): 1 year
- Call option
- Option strike price (European type): 95

1. The random numbers are between 0 and 1 : first column of the confidence level / probability table.

2. The first simulation gives a value of 0.75. The number of standard deviations above the mean so that 75% of the variables are located to the left of the normal law curve and 0.6745 standard deviations (see column of number of standard deviations) . The second 0.5 simulation that corresponds to average.

3. The change in price is calculated as follows: average yield (10%) plus the number of standard deviations multiplied by the standard deviation (20%). Thus, for the first joint we have the variation: 0.1 + 0.2 * 0.6745 = 0.2349. This value can be obtained directly through the function NORMINV.

4. This variation can be transformed into an exponential variation (the exponential of (0.2349) corresponds to 1.26478).

5. Price of the stock with variation will result from the product of the exponential variation by the original: 1.26478 * 100 = 126.478. The other prices are calculated in the same way.

6. Intrinsic value of the option and calculated by the difference between the share price with variation and the exercise price: 126.478 - 95 = 31.478. When this difference is negative, the value is zero, because in this case the option will not be exercised (the exercise price is higher than the share price).

7. The option value is updated, because it will only be exercised within 1 year (European option). The update is carried out at a continuous interest rate or exponential that assumes that the rate is composed continuously. Thus, for a 10% reference rate, the update factor will be 1 / exp (0.1) = 0.9048

8. Next, an average of the current value of the option is calculated, that is, 21.06 for an average share price of 116.62. It is also possible to estimate the number of times that the intrinsic value of the option will be positive and, therefore, exercised, in our example, will be 60% (6 times out of 10)

Table 5: Monte Carlo simulation calculation: 1st generation of random numbers

Share price: 100
Standard deviation: 20%
Yield rate: 10%
Time period: 1

Probability (a)	N° of standard desviations (b)	Rate of return	Standard desviation	Variation (c)	Use of the normin function	Exponential variation	Price of the share	Price of share with variation	Exercise price	Value intrinsic of the option	Factor update continuous 1 / ex (0,1)	Current value option
						Exp(var)						
							8	9	10	11=9-10	12	13=11*12
0,75	0,67448975	0,1	0,2	0,23489795	0,23489795	1,264779691	100	126,4779691	95	31,47796914	0,904837418	28,48244432
0,5	0	0,1	0,2	0,1	0,1	1,105170918	100	110,5170918	95	15,51709181	0,904837418	14,04044529
0,2	-0,841621234	0,1	0,2	-0,0683242	-0,0683242	0,933957592	100	93,39575918	95	-1,604240821	0,904837418	0
0,1	-1,281551566	0,1	0,2	-0,1563103	-0,1563103	0,85529374	100	85,52937403	95	-9,470625967	0,904837418	0
0,85	1,036433389	0,1	0,2	0,30728668	0,30728668	1,359730717	100	135,9730717	95	40,97307169	0,904837418	37,0739684
0,79	0,806421247	0,1	0,2	0,26128425	0,26128425	1,298596738	100	129,8596738	95	34,85967383	0,904837418	31,54233726
0,22	-0,772193214	0,1	0,2	-0,0544386	-0,0544386	0,947016613	100	94,70166133	95	-0,298338667	0,904837418	0
0,95	1,644853627	0,1	0,2	0,42897073	0,42897073	1,535676077	100	153,5676077	95	58,56760774	0,904837418	52,99416297
0,92	1,40507156	0,1	0,2	0,38101431	0,38101431	1,463768555	100	146,3768555	95	51,37685548	0,904837418	46,48770126
0,15	-1,036433389	0,1	0,2	-0,1072867	-0,1072867	0,898268122	100	89,82681225	95	-5,173187754	0,904837418	0
				Average share price				116,6225876				21,06210595

Source: authors elaboration

In 60% of the time the option is positive

(a) Using rand () usage. Generation of random numbers between 0 and 1.
(b) Inverse of the normal distribution function: normsinv function number of standard deviation.
(c) Thus, for a 95% confidence interval, we will have a volatility equivalent to 1.645 standard deviations from the inverse mean of the cumulative normal to mean distribution and a specific standard deviation: normsinv.

The VAR would be calculated by ordering the current values of the option, as follows:

Table 6 - ordered values

Probability	Sorted	Loss ordered VAR
0.1	52.99415	-52.9941
0.2	46.48776	-46.4878
0.3	37.07396	-37,074
0.4	31.54235	-31.5423
0.5	28.48246	-28.4825
0.6	14.04045	-14.0404
0.7	0	0
0.8	0	0
0.9	0	0
1	0	0

Source: authors elaboration

Note: The probability column is calculated considering that each observation represents 10% (1 in 10 observations in total, given that only 10 are considered). In this case, 40% do not exercise the option, losing the respective prize. In our example, no price was considered for the option premium.

However, if we used another generation of random numbers, the result would be different.

Table 7: Monte Carlo simulation calculation: 2nd generation of random numbers

Probability (a)	N° of standard desviations (b)	Rate of return	Standard desviation	Variation (c)	Use of the normin function	Exponential variation	Price of the share	Price of share with variation	Exercise price	Value intrinsic of the option	Factor update continuous 1 / ex (0,1)	Current value option
1	2	3	4	5 = 3 + 4 * 2	6	7 = exp (6)	8	9 = 7 * 8	10	11 = 9 - 10	12	13 = 11 * 12
0,26733781	-0,6208845	0,1	0,2	-0,0241769	-0,0241769	0,97611302	100	97,611302	95	2,611302	0,904837418	2,362803761
0,61208833	0,28476611	0,1	0,2	0,156953221	0,15695322	1,16994088	100	116,9940885	95	21,9940885	0,904837418	19,90107421
0,24631463	-0,686133	0,1	0,2	-0,03722659	-0,0372266	0,9634578	100	96,34577963	95	1,34577963	0,904837418	1,217711765
0,40672037	-0,2359896	0,1	0,2	0,05280208	0,05280208	1,05422097	100	105,4220973	95	10,4220973	0,904837418	9,430303609
0,63620766	0,34834024	0,1	0,2	0,169668047	0,16966805	1,18491145	100	118,4911451	95	23,4911451	0,904837418	21,2556671
0,08628703	-1,3639794	0,1	0,2	-0,17279587	-0,1727959	0,84130933	100	84,13093312	95	-10,869067	0,904837418	0
0,41206343	-0,2222403	0,1	0,2	0,055551949	0,05555195	1,05712393	100	105,7123933	95	10,7123933	0,904837418	9,692974254
0,99095823	2,36390314	0,1	0,2	0,572780627	0,57278063	1,77319079	100	177,3190785	95	82,3190785	0,904837418	74,48538248
0,26227264	-0,6363547	0,1	0,2	-0,02727093	-0,0272709	0,97309756	100	97,30975608	95	2,30975608	0,904837418	2,089953729
0,73597912	0,63099811	0,1	0,2	0,226199622	0,22619962	1,25382593	100	125,3825932	95	30,3825932	0,904837418	27,49130715
				Avarage share price				112,4719167	Price		Avarage price	16,79271781

Source: authors elaboration

In 90% the value is positive

In practice, Monte Carlo simulations imply:

1. Countless of parameters must be stipulated, that is, among others: the interest rate, volatilities, prices of the underlying assets, which leads to the number of simulations is high, in order to consider variations in these parameters. The number of simulations can rise to large thousands;
2. The variations change continuously;
3. The correlations between the different instruments must also be considered.

Once these calculations have been made, taking into account these adjustments, it is enough to order the results of the simulations and consider the lowest values that represent 5% of the set, in order to obtain the VAR with a 95% confidence level.

3.3 Advantages and disadvantages of Monte Carlo methodologies

Although this method is more accurate than the previous historic simulation for situations unpredictable (cases where history does not show consistent behavior), when considering countless situations, necessarily implies substantial computer resources. However, computer resources have become increasingly accessible, given technological progress. On the other hand, some simplifications can be considered in order to reduce the number of simulations. This method is particularly suitable for portfolios that contain complex instruments, including exotic options.

IV. CONCLUSIONS ON VAR

The VAR must be considered a concept and not a calculation system. The reasons for its popularity have to do with the following:

• It focuses on one of the main types of concerns for decision-makers and top managers - the potential for significant losses;

• Due to the relevance of its meaning, it won the support of important institutions and bodies, namely, the European Union and the BIS (Basel II Agreement)

VAR is a statistical methodology that helps managers to add the value of the risks related to business and product lines in an intelligible way. This helps financial institutions to impose limits on exposures to market risk and to optimize the allocation of capital to various businesses or portfolios.

In this way, VAR is an important tool that helps managers to have an aggregate view on the institution's risk profile and to control risk by imposing limits. VAR provides simple added value that can even help supervisors to impose prudential limits on each institution.

Thus, VAR's main users are Financial Institutions and Multinational companies, which hold investment portfolios.

However, the VAR contains a significant level of subjectivity, namely in the definition of the level or degree of confidence, the time horizon and, essentially, the calculation method. On the other hand, it should not be applied to illiquid instruments and the value of the portfolios must be able to be adjusted in relation to the time horizon.

Accordingly, in normal markets, it provides a powerful instrument, as well as for short forecast periods, but presents difficulties when markets are turbulent or when the forecast horizon is long.

In this way, VAR models should be complemented, among other aspects, with adequate backtesting programs (which assess the adequacy of the model) and stress tests (which incorporate the assessment of the impact of extreme scenarios, although plausible).

In turn, the steps for measuring the VAR go through:

1. Characterization of the risk exposures of the positions taken. As examples of the type of exposures (key vectors), we have: shares, bonds, raw materials, commodities, foreign exchange, interest rates, implied volatilities, margins or others;

2. Characterization of uncertainty, taking into account the assessment of the situation and evolution of the markets. The characterization of uncertainty consists in determining the joint probability distribution of the key vector, called the inference process. For this purpose, historical data can be used or techniques of analysis of chronological succession can be used to characterize the distribution conditioned to information existing in the initial time;

3. Combining the characteristics of the previous two steps to value the portfolio's market risk through a metric VAR (measurement methodology).

V. ENDNOTES

[1] Of the characteristics of the methods of Historical Simulation and Monte Carlo, we highlight the fact that both involve the complete revaluation of the portfolio (full valuation). The complete revaluation requires the calculation of the portfolio value for various price levels, the variation in the portfolio value being obtained by the difference between the estimated value for the next period and the current value of the portfolio. On the contrary, in the VAR estimation method called "local valuation" (analytical method), the change in the portfolio's value is estimated directly from the estimation of the sensitivity of the portfolio's value to changes in asset prices.

[2] It involves identifying the market variables that constitute exposures, such as: stocks, bonds,

commodities, raw materials and goods, foreign exchange, interest rates, implied volatilities, margins or others. Banco de Portugal Instruction No. 4/2008 mentions the following risk categories: general position risk in debt instruments; specific position risk in debt instruments; general risk of equity securities positions; specific position risk in equity securities, foreign exchange risk and commodities risk.

[3] Pseudo random numbers are designated because they are obtained through a deterministic process.

[4] Afonso, António and others on page 306 state "The strong hypothesis that the expected return on financial assets follows normal distributions, can skew the results. Usually the value of the VAR is very sensitive to the values of the distribution tabs, particularly the distribution probability in the left tab (that models potential losses.) Indeed, the returns on financial assets are usually leptocurtic, that is, they exhibit a higher proportion of extreme observations than would be expected in the case of a normal distribution.

Extreme situations of volatility in the financial markets, where the potential losses of the market value of the assets could hardly be anticipated, even with the calculation of the VAR, were, for example, the stock market “crash” of October 19, 1987, when the index stock exchange S&P 500 varied by 22.3 standard deviations, or even when on January 8, 1988 the same index recorded a variation of 6.8 standard deviations ". Note on the meaning of leptokurtic: In descriptive statistics, kurtosis is a measure of dispersion that characterizes the “flattening” of the distributions function and given by the following expression.

m4 (μ) / σ^4 – 3 with 3 being normal kurtosis

m4 (μ) fourth central moment σ standard deviation

If the kurtosis value = 0, then the distribution has the same flatness as the normal distribution. This function is called mesocurtic;

If the kurtosis value > 0 then the distribution is higher (tapered) and concentrated than the normal distribution. This function is called leptocurtic, or that has heavy tails (the meaning that it is relatively easy to obtain values that deviate from the average several multiples of the standard deviation); multiples of the standard deviation);

If the kurtosis value <0 then the distribution function is flatter than the distribution normal. This function is called platicurtica.

[5] The use of exponential moving averages (exponential decay factor) mitigates this issue. The latest observations carry more weight than the older ones.

[6] Basically, we have two methods: analytical methods and Monte Carlo simulation methods. The analytical methods represent the system by an analytical model and calculate the indices using mathematical solutions. The flexibility of the Monte Carlo simulation allows the generation of scenarios for wide range events and estimates the indices by simulation. Analytical methods solve the equations used to describe the models.

[7] Random numbers are called pseudo numbers when they are obtained through a deterministic process. A deterministic process is the opposite of a random process. A random and governed process for un-predictability. Such a process produces different and unpredictable results each time it is run.

[8]In probability, a random variable X has a lognormal distribution when its logarithm $Y = log\ (X)$ has a normal distribution. For example, the price of a stock in the future can be modeled as the effect of several small independent adjustments.

[9] The buyer (holder) of a call option in relation to a specific underlying asset acquires the right, but not the obligation, to buy that asset, at a previously established price (strike price) and on a specific date, paying for this right a premium to the option seller. Consequently, the seller (writer or seller) assumes the obligation to sell the respective asset and under the agreed conditions, if the buyer exercises his right.
[10] The EXCEL spreadsheet allows you to generate such numbers, see the rand () function.

[11]The standardized (or reduced) normal distribution and a normal distribution with a mean of zero and standard deviation equal to 1 (ie, μ = 0 and σ = 1). Any normal distribution can be converted to a standardized normal distribution, making μ = 0 and expressing deviations from μ in standard deviation units (z scale). Under such conditions, 68.26% of the area (probability) under the standardized normal curve and included within an average standard deviation, that is, within μ +/- 1 σ, 95.54% within μ+/- 2 σ, and 99.74% withinμ +/- 3 σ

[12] In EXCEL the function that gives us this information is the normsinv function, which consists of the inverse of normal distribution function.

[13] Inverse of the cumulative normal distribution for a specific mean and standard deviation

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Annex 1: Comparison of the three methodologies / approaches

- Variance / Covariance
- Historical Simulation
- Monte Carlo Simulation

Comparison of the three VAR methodologies

Approach Variance - Covariance

Benefits	Disadvantages
Computationally efficient. It takes a short time to calculate a bank's position. Due to the central limit theorem, the methodology can be applied, even if the risk factors do not have a normal distribution.	Assumes normal distribution. It assumes that the risk factors follow a multivariate log-normal distribution. It does not treat well when the distributions are long ("fat-tailed") - extreme events.

Approach Variance – Covariance

Benefits	Disadvantages
No pricing model is required, only Greeks are needed, and these can be supplied directly by most systems already installed in banks. Incremental VAR is easy to handle.	It requires estimating the volatility of risk factors, as well as the correlations of their returns. Bond returns can be approximated by Taylor's expansion. However, in some circumstances, second-order expansion may not be sufficient, when we are faced with exotic options it cannot be used to conduct sensitivity analyzes.

Approach - Historical Simulation

Benefits	Disadvantages
It is not necessary to make any assumptions about the distribution of risk factors. It is not necessary to estimate volatilities and correlations. They are implicitly captured by the database used. Long distributions and other external events are captured as long as they are in the database used.	Complete dependence on historical data and its particularities. In this way, extreme cases can be ignored if they do not appear in the database, or, instead, they can distort if they do not accommodate changes in the market structure, such as the introduction of the euro.

Approach - Historical Simulation

Benefits	Disadvantages
Aggregation across markets is straightforward. It allows the calculation of the confidence interval for VaR.	Sparse data can lead to skewed VAR results. It cannot be used to conduct sensitivity analyzes. It is not computationally enough when we are dealing with portfolios with complex products.