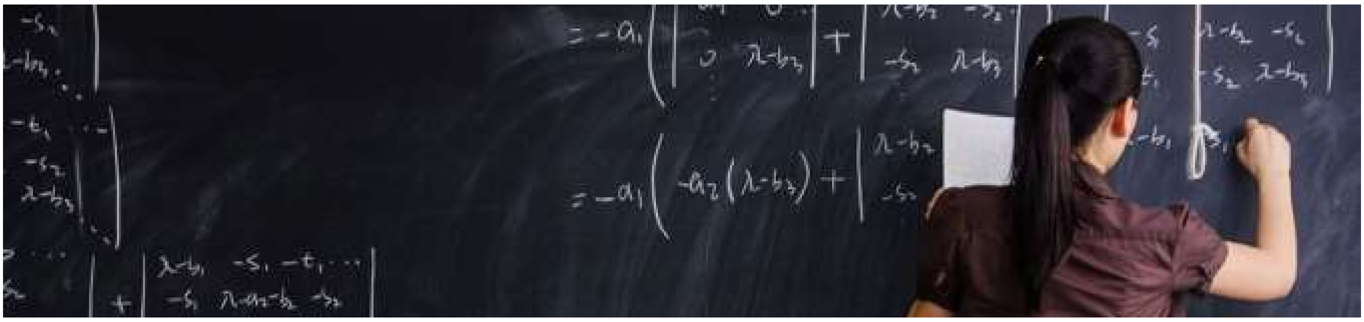
A MATHEMATICAL CLOSURE MODEL : Arrest of a crack in Infinite Slit in Elastic Plastic Media



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ABSTRACT

In this paper, the problem of a finite hairline straight crack weakening an infinite homogeneous elastic perfectly-plastic plate is considered. The plate is subjected to uniform constant unidirectional tension which is applied to it at infinite boundary. Consequently, face of crack open in mode-I type deformation farming plastic Zones ahead of the tip of the crack. To stop the crack form further extension rims of the plastic zones are subjected to normal cohesive cubically varying yield point stress distribution. To visualize the entire physical picture of the problem, problem is considered in two components which are superimposed afterwards to get the solution of original problem. Wester Gard technique is problem. Wester Gard technique is applied to solve the problem. Variation of load required for the closure of plastic zones is studied with respect to the affecting parameters. Results obtained are analyzed and graphically reported.

Keywords :

Mode I type deformation,
rims of the plastic zones,
Wester Gard technique,
Yield point stress distribution.

INTRODUCTION

In the present paper, an infinite homogenous elastic perfectly plastic plate weakened by a finite hairline straight crack is subjected to uniform Constant tension at infinite boundary. This results in the formation of plastic zones ahead of tips of crack. These plastic zones are closed by normal cohesive cubically varying yield point stress distribution. Principal of superposition and wester Gard hypothesis are used to obtain the analytic solution of problem.

Perkins explained environmental effects in crack growth. He found that interaction between materials and environment can result in slow crack growth at stress intensities markedly below those associated with fast fracture. Dug dale in his model tested the closure of internally cracked plate specimen under tensile conditions which opens the rims of the crack tip are them closed by normal cohesive uniform constant yield point stress distribution. Dug dale model solution for two collinear unequal hairline straight cracks was obtained by Bhargava, Agarwal and Hasan Efits and Liebowitz worked on modified wester Gard equations for Certain plane crack problems.

Fundamental Equations: - The general method of obtaining the solution of the Dug dale model with an arbitrary power of stress distribution in the plastic zone follows. The stress function due to the loading in the plastic zones is found form the equation given below.

$$\phi(z) = \frac{2}{\pi} \sum_{j=0}^{j=n} C_j \int_a^b \frac{t^j z \sqrt{b^2-t^2}}{(z^2-t^2)\sqrt{z^2-b^2}}\, dt,$$

When C_j is constant and b – a denotes the plastic zone size……………… (1)

The singular components of Ø (z) are set equal and opposite to the singular stress function due to remote loading. A relation for the remote applied stress Obtained from this equality. The net stress function then becomes.

Ø (z) = non-singular terms of

$$\left(\frac{2}{\pi} \sum_{j=0}^{n} C_j \int_a^b \frac{t^j z \sqrt{b^2-t^2}}{(z^2-t^2)\sqrt{z^2-b^2}}\, dt \right)$$

Statement of the problems: - AN infinite elastic perfectly plastic plate occupying XOY – plate is weakened by a finite straight crack L: [-a, a]. Uniform constant Unidirectional tension $P_{yy}{}^{\pm} = \sigma_\infty P_{xy} \pm = o$ is applied at infinite boundary parallel to Y- axis This causes opening of the face of crack farming plastic zones ahead of the tip of crack . Plastic zones Γ_1, occupies the interval [-b,-a] While the interval [a, b] is occupied by the plastic zone Γ_2. Rims of plastic zones Γ_i (i=1, 2) are subjected to stress distribution $P_{yy}{}^{\pm} = t^3\ \sigma_{ye}$, $P_{xy}{}^{\pm} = 0$. The configuration of the problem is depicted in figure 1.

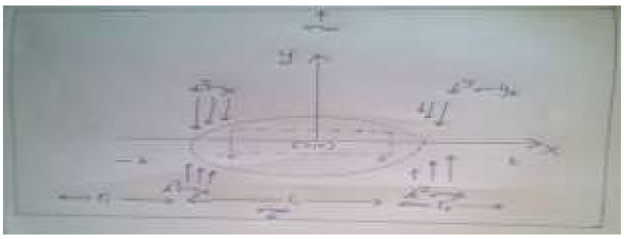

Figure 1: Configuration of the Problem

Solution of the problem: - Solution of the problem is obtained by superimposing the solution of two component problems, appropriately derived from the original problem. These problems are termed as problem I and problem II.

Statement and solution of problem I: An infinite elastic perfectly plastic plate is weakened by a finite crack L: [-a, a]. Uniform constant unidirectional tension $P_{yy}\pm = \sigma_\infty$ $P_{xy} \pm = 0$ applied at infinite boundary of the plate opens the face of crack farming plastic zones ahead of crack tips. This can be altered to the case of Uniaxial tensile loading by superposition of the stress $P_{yy}{}^{\pm} = -\sigma_\infty$, $P_{xy}{}^{\pm} = 0$. The westergaard type stress function for the problem is given by

$$\phi_1(z) = \frac{\sigma_\infty z}{\sqrt{z^2-b^2}},$$

Where Z = x + i y -- (2)

Statement and Solution of problem II: - An infinite elastic perfectly – plastic plate is weakened by a straight crack L as defined in problem I. Plastic zones farmed due to unidirectional tension $Pyy^{\pm} = \sigma_\infty$ $P_{xy}{}^{\pm} = 0$ are arrested by applying cubically varying yield point stress distribution $P_{yy}{}^{\pm} = t^3\sigma ye$, $P_{xy}{}^{\pm} = 0$ at the rims of crack . The stress function for above forces is given by

$$\phi_{II}(z) = \frac{2}{\pi}\left(\frac{\sigma_{ye}\, z \sqrt{b^2-t^2}}{(z^2-t^2)\sqrt{z^2-b^2}} \right)$$

Where 2b is the length of the extended crack at any point 't'. Thus the stress function due to the tensile Tractions on the plastic zones using equation (i) will be

$$\phi_{II}(z) = \int_a^b \frac{2}{\pi} \frac{t^3 \sigma_{ye}\, z \sqrt{b^2-t^2}}{(z^2-t^2)\sqrt{z^2-b^2}}\, dt$$

On solving above equation one get

$$\phi_{II}(z) = \frac{2}{\pi}\sigma_{ye} z \left[\frac{\sqrt{b^2-a^2}}{3} \cdot \frac{(2b^2+a^2)}{\sqrt{z^2-b^2}} + \sqrt{z^2-b^2}\sqrt{b^2-a^2} - z^2 \cot^{-1}\frac{\sqrt{z^2-b^2}}{\sqrt{b^2-a^2}} \right] \text{---} (3)$$

Hence the total stress function for the Original problem is depicted by the principle of super position using equations 2 and 3 and is given by

$$\phi_R(z) = \phi_1(z) + \phi_{II}(z)$$

$$\text{i.e. } \phi_R(z) = \frac{\sigma_\infty z}{\sqrt{z^2-b^2}} - \frac{2}{\pi}\sigma_{ye} z \left[\frac{\sqrt{b^2-a^2}}{3} \frac{(2b^2+a^2)}{\sqrt{z^2-b^2}} + \sqrt{z^2-b^2}\sqrt{b^2-a^2} - z^2 \cot^{-1}\frac{\sqrt{z^2-b^2}}{\sqrt{b^2-a^2}} \right] \text{---} (4)$$

Plastic zone length: - Both $\text{Ø}_I(Z)$ $\text{Ø}_{II}(Z)$ Given rise to stress singularities at (b,o). Thus the stress intensity factor K at the tip X_x=b for original problem using wester gaard function is given by

$$(K)_b = \lim_{|\epsilon| \to 0} \sqrt{2\pi\epsilon} \cdot \phi_R(z) \text{ ---} (5)$$

Substituting the value of $\varnothing_R(Z)$ from equation (4) in equation (5) one get

$$(K)_b = \sigma_\infty \sqrt{\pi b} - \frac{2}{a}\sigma_{ye}\frac{\sqrt{b(b^2-a^2)}}{\pi}\left(2b^2+a^2\right) \quad \text{---(6)}$$

Since stress remain finite at every point of the problem $(K)_b$ is equal to zero thus one gets

$$\frac{\sigma_\infty}{\sigma_{ye}} = \frac{2}{\pi}\frac{\sqrt{b^2-a^2}}{3}\left(2b^2+a^2\right) \quad \text{---(7)}$$

The plastic zone length (b-a) is then determined using above equation.

Case study: - Figure 2 depicts the behavior of load ratio (load applied at infinity to yield point stress) required to close the plastic zones development versus the ratio of length of plastic zone tip and crack tip from Origin. It may note that as a/b increases load ratio also increases.

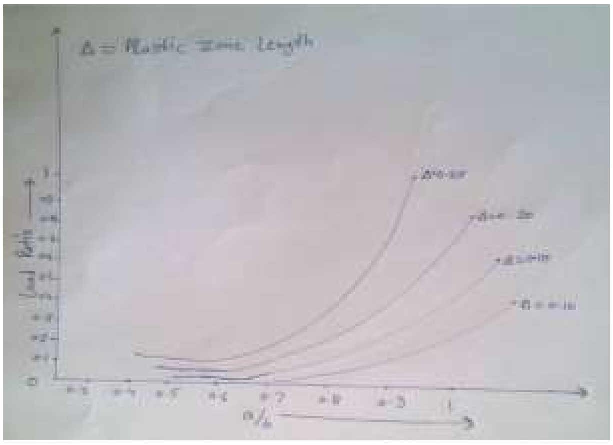

Figure 2 – Variation of required load ratio of the lengths crack tip and plastic zone tip from origin

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